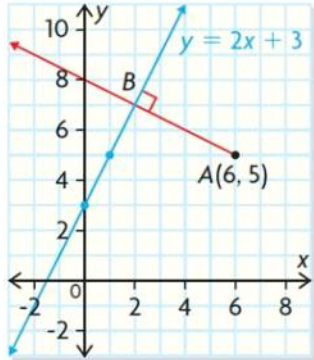# Distance between a Point and a Line

Recall the length of a line segment formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Calculate the shortest distance between the point $A(6, 5)$ and the line $y = 2x + 3$.

The shortest distance is the line segment connecting the point and the line such that the segment is perpendicular to the line.

1. Determine the equation of the line passing through $A(6, 5)$ and perpendicular to the line $y = 2x + 3$.

$$y = -\tfrac{1}{2}x + b$$
$$5 = -\tfrac{1}{2}(6) + b$$
$$5 = -3 + b$$
$$8 = b \qquad \Rightarrow \quad y = -\tfrac{1}{2}x + 8$$

2. Solve the system of equations.

$$\left. \begin{array}{l} y = 2x + 3 \\ y = -\tfrac{1}{2}x + 8 \end{array} \right\} \quad (2x + 3 = -\tfrac{1}{2}x + 8) \times 2$$

$$4x + 6 = -x + 16$$
$$+x \qquad\qquad +x$$
$$5x + 6 = 16$$
$$ -6 \quad -6$$
$$5x = 10$$
$$x = 2 \quad \Rightarrow \quad y = 2(2) + 3 = 7$$
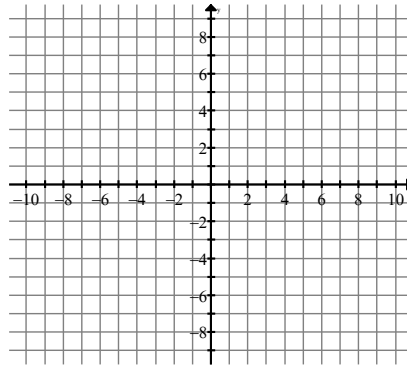$$(2, 7)$$

3. Calculate the distance between the points.

$$(6, 5) \text{ and } (2, 7)$$

$$d = \sqrt{(6-2)^2 + (5-7)^2}$$
$$= \sqrt{4^2 + (-2)^2}$$
$$= \sqrt{16 + 4} \; = \sqrt{20} \; \doteq 4.47 \text{ units}$$

Calculate the shortest distance between the point G(-4, 4) and the line $y = 3x - 4$.

1. Determine the equation of the line passing through (-4, 4) and perpendicular to $y = 3x - 4$.

$$y = -\frac{1}{3}x + b$$

$$4 = -\frac{1}{3}(-4) + b$$

$$4 = \frac{4}{3} + b$$

$$-\frac{4}{3} \qquad -\frac{4}{3}$$

$$\frac{12}{3} - \frac{4}{3} = b$$

$$\frac{8}{3} = b \quad \rightarrow \quad y = -\frac{1}{3}x + \frac{8}{3}$$

2. Solve the system of equations.

$$\left.\begin{array}{l} y = 3x - 4 \\ y = -\frac{1}{3}x + \frac{8}{3} \end{array}\right\}$$

$$\left(3x - 4 = -\frac{1}{3}x + \frac{8}{3} \right) \times 3$$

$$9x - 12 = -x + 8$$

$$10x = 20$$

$$x = 2$$

$$y = 3(2) - 4 = 2$$

3. Calculate the distance between (-4, 4) and (2 , 2).

$$d = \sqrt{(-4-2)^2 + (4-2)^2}$$

$$= \sqrt{(-6)^2 + (2)^2}$$

$$= \sqrt{36 + 4} = \sqrt{40} \doteq 6.32 \text{ units}$$

Assignment:
Calculate the distance between the given line and point. Round your answer to one decimal place.

1. $y = 4x - 2$; (-3, 3)
2. $y = -x + 5$; (-1, -2)
3. $2x + 3y = 6$; (7, 6)