

U6 Practice Test

Pre-Calc 12

Geometric Sequences and Series Practice Test

Name: _____

/40

$t_n = ar^{n-1}$	$S_n = \frac{a(1-r^n)}{1-r}$	$S_\infty = \frac{a}{1-r}$
------------------	------------------------------	----------------------------

1. Determine t_{12} of the geometric sequence $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \dots$ [2]

$$t_{12} = \frac{1}{6}(2)^{11} = \frac{1024}{3} \text{ or } 341.\bar{3}$$

you don't need to write both!

2. Determine the tenth term of the geometric sequence 2187, -729, 243, ... [2]

$$r = \frac{-729}{2187} = -\frac{1}{3}$$

$$t_{10} = 2187\left(-\frac{1}{3}\right)^9 = -\frac{2187}{19683} = -\frac{1}{9} \text{ or } -0.\bar{1}$$

3. In a geometric sequence, $t_8 = 6561$ and the common ratio is $\frac{3}{2}$. Determine the value of t_1 . [2]

$$t_1 \left(\frac{3}{2}\right)^7 = 6561$$

$$t_1 = 6561 \div \left(\frac{3}{2}\right)^7$$

$$= 6561 \cdot \left(\frac{2}{3}\right)^7$$

$$= 384$$

4. Which term of the geometric sequence 3, 6, 12, ... is 786432? [2]

$$3(2)^{n-1} = 786432$$

$$\div 3 \qquad \div 3$$

$$2^{n-1} = 262144$$

$$2^{n-1} = 2^{18}$$

$$\therefore n-1 = 18 \rightarrow n = 19$$

5. In a geometric sequence, $t_3 = 18$ and $t_6 = -486$. Determine the eighth term. [3]

$$18r^3 = -486$$

$$\div 18 \qquad \div 18$$

$$r^3 = -27$$

$$r = -3^*$$

*only one possible answer

$$t_8 = -486(-3)^2 = -4374$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

6. Determine the sum of the first 10 terms of the geometric series $16 - 8 + 4 - \dots$ [2]

$$\begin{aligned} S_{10} &= \frac{16(1-(-\frac{1}{2})^{10})}{1-(-\frac{1}{2})} \\ &= 10.65625 \\ &\text{or } \frac{341}{32} \end{aligned}$$

7. Evaluate $\sum_{k=1}^6 (-5)(2)^{k-1}$. [2]

$$\begin{aligned} S_6 &= \frac{-5(1-2^6)}{1-2} \\ &= -315 \end{aligned}$$

8. Determine the sum from t_5 to t_{12} of the geometric series $648 - 216 + 72 - \dots$ [2]

$$\begin{aligned} S_{12} - S_4 &= \frac{648(1-(-\frac{1}{3})^{12})}{1-(-\frac{1}{3})} - \frac{648(1-(-\frac{1}{3})^4)}{1-(-\frac{1}{3})} \\ &= 5.99908550\dots \end{aligned}$$

9. Determine the sum of the terms $54 - 18 + 6 - \dots + \frac{2}{27}$. [3]

$$54\left(-\frac{1}{3}\right)^{n-1} = \frac{2}{27}$$

$$\left(-\frac{1}{3}\right)^{n-1} = \frac{1}{729}$$

$$\left(-\frac{1}{3}\right)^{n-1} = \left(-\frac{1}{3}\right)^6$$

$$\therefore n-1 = 6$$

$$n = 7$$

$$S_7 = \frac{54(1-(-\frac{1}{3})^7)}{1-(-\frac{1}{3})}$$

$$= \frac{1094}{27}$$

$$\text{or } 40.\overline{518}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

10. Find the sum of each infinite geometric series, if possible. [3]

a) $45 - 30 + 20 - \dots$

$$r = \frac{-30}{45} = -\frac{2}{3}$$

$$S_\infty = \frac{45}{1 - (-\frac{2}{3})} = 27$$

b) $\frac{1}{18}, \frac{1}{12}, \frac{1}{8}, \dots$

$$r = \frac{\frac{1}{12}}{\frac{1}{18}} = \frac{18}{12} = \frac{3}{2} > 1$$

not possible

11. Determine the sum of the geometric series $\sum_{k=1}^{\infty} 2 \left(\frac{1}{3}\right)^{k-1}$. [2]

$$\begin{aligned} S_\infty &= \frac{2}{1 - \frac{1}{3}} \\ &= \frac{2}{\frac{2}{3}} \\ &= 3 \end{aligned}$$

12. The sum of an infinite geometric series is 18. If the common ratio is $-\frac{1}{3}$, determine t_1 . [2]

$$\begin{aligned} \frac{t_1}{1 - (-\frac{1}{3})} &= 18 \\ \frac{\frac{4}{3} \cdot t_1}{\frac{4}{3}} &= 18 \cdot \frac{4}{3} \\ t_1 &= 24 \end{aligned}$$

13. Use an infinite geometric series to express $0.\overline{132}$ as a fraction. [3]

$$\begin{aligned} 0.\overline{132} &= 0.1 + 0.032 + 0.00032 + 0.0000032 + \dots \\ &= 0.1 + \sum_{k=1}^{\infty} 0.032 (0.01)^{k-1} \\ &= 0.1 + \frac{0.032}{1 - 0.01} \\ &= 0.1 + \frac{0.032}{0.99} \\ &= \frac{99}{990} + \frac{32}{990} \\ &= \frac{131}{990} \end{aligned}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

14. Is the series $5 + 4 + 3.2 + \dots$ convergent, or divergent? Justify your answer. [2]

$$r = \frac{4}{5}$$

Since $-1 < r < 1$, the series is convergent.

15. A shoe store is closing and wants to sell all its shoes. At the beginning of each week, the price of all shoes is reduced by 10% of the current price. If a pair of shoes costs \$100.00 during the first week of the sale, determine the price of these shoes during the 8th week of the sale. [3]

$$t_1 = 100$$

$$t_2 = 100(0.9)$$

$$t_3 = 100(0.9)^2$$

\vdots

$$t_8 = 100(0.9)^7 = 47.83$$

The shoes are \$47.83 during the 8th week.

16. A ball is dropped from a height of 3 m. After each bounce, the ball rises to 60% of its previous height.

a) To the nearest centimeter, to what height does the ball ^{rise} after the 4th bounce? [2]

$$t_1 = 3(0.6) = 1.8$$

$$3(0.6)^2 \quad t_2 = 1.8(0.6)$$

\vdots

$$t_4 = 1.8(0.6)^3 = 0.3888$$

The ball rises 0.39m (or 39 cm).

b) What is the total vertical distance that the ball travels (up and down) before it comes to rest? **Include the initial 3 m drop** in your final answer. [3]

$$t_1 = 1.8 \cdot 2 = 3.6 ; r = 0.6$$

$$\begin{aligned} & 3 + S_\infty \\ & = 3 + \frac{3.6}{1-0.6} \end{aligned}$$

$$= 3 + \frac{3.6}{0.4}$$

$$= 3 + 9$$

$$= 12$$

The total vertical distance is 12m.