Recall: Given $y = af\left(k(x-d)\right) + c$

  ✓ a: vertical stretch/compression and reflection across the x-axis if $a < 0$
  ✓ k: horizontal stretch/compression and reflection across the y-axis if $k < 0$
  ✓ d: horizontal translation
  ✓ c: vertical translation

Given $y = a\sin\left(k(\theta - d)\right) + c$, we say that a affects the **amplitude**, k affects the **period**, and d affects the **phase shift**.

- Changing units to radians will not affect vertical transformations (amplitude & vertical translation)
- d will now be expressed in radians
- The period of $y = \sin x$ and of $y = \cos x$ is 2π

  ∴ the period of a transformed sinusoidal function $= \dfrac{2\pi}{k}$
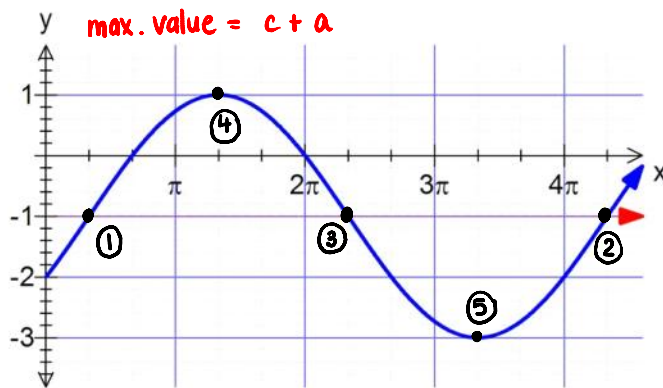
Ex. Sketch the graph of $y = 2\sin\left(\dfrac{1}{2}\theta - \dfrac{\pi}{6}\right) - 1$ over one full period.

Rewrite the equation by factoring out the value of k: $y = 2\sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{3}\right)\right) - 1$

- Amplitude =   2

- Period = $\dfrac{2\pi}{1/2} = 4\pi$

- Phase shift* = $\dfrac{\pi}{3}$ to the right   *Note: a positive value indicates movement to the right; a negative value indicates movement to the left

- Vertical translation = -1
            or 1 unit down

min. value $= c - a$
max. value $= c + a$

③ half way between ① and ②

④ half way between ① and ③ at max. value

⑤ half way between ③ and ② at min. value

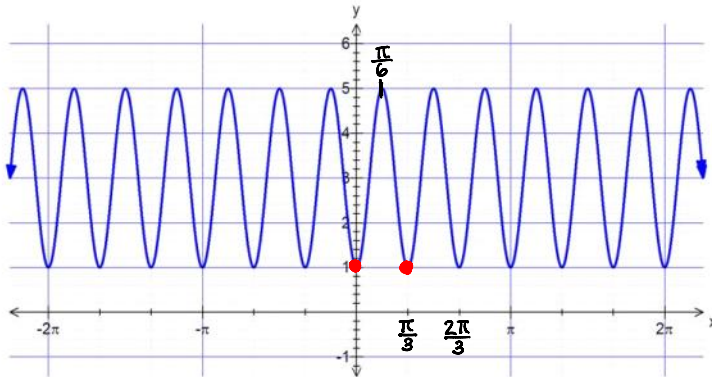Ex. Write the equation of a sine function with an amplitude of 3 and a period of π.

$$\text{period} = \frac{2\pi}{k}$$

$$\pi = \frac{2\pi}{k}$$

$$k = \frac{2\pi}{\pi} = 2$$

$$\therefore y = 3\sin 2\theta$$

Ex. Determine two possible cosine equations for the following graph:

$$\text{amp.} = \frac{\text{max.} - \text{min.}}{2}$$

vert. translation :

$$\frac{\text{max.} + \text{min.}}{2}$$

- Amplitude $= \frac{5-1}{2} = 2$
- Period $= \frac{\pi}{3} \quad \therefore k = \frac{2\pi}{\pi/3} = 6$
- Phase shift/reflection across x-axis...
- Vertical translation $= \frac{5+1}{2} = 3$

Two possible equations are:

$$y = -2\cos(6\theta) + 3$$

$$y = 2\cos\left(6\left(\theta - \frac{\pi}{6}\right)\right) + 3$$

$$y = 2\cos\left(\frac{\pi}{4}(\theta - 6)\right)$$

$$y = -2\cos\left(\frac{\pi}{4}(\theta - 2)\right)$$

$$y = 2\sin\left(\frac{\pi}{4}\theta\right)$$
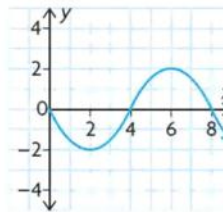
Ex: Determine a possible equation for the graph shown.

amplitude $= 2$
period $= 8 \quad \therefore k = \frac{2\pi}{8} = \frac{\pi}{4}$
reflection in x-axis
vertical translation: none

$$y = -2\sin\frac{\pi}{4}\theta$$

Assignment: p. 343 #1d, 4c, 8cf, 10, 14c