Practice Test: Polynomials

PC 12 **Polynomial Expressions & Functions Practice Test** Name: _____

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1. Calculate $(2x^3 + 5x^2 - 4x - 5) \div (x + 2)$ using long division. Write the division statement.

$$\begin{array}{r} 2x^2 + x - 6 \\ x+2 \overline{)\,2x^3+5x^2-4x-5} \\ \underline{2x^3+4x^2} \\ x^2-4x \\ \underline{x^2+2x} \\ -6x-5 \\ \underline{-6x-12} \\ 7 \end{array}$$

2 marks

$2x^3 + 5x^2 - 4x - 5 = (x+2)(2x^2+x-6)+7$

2. Calculate $(x^2 - 15x + 2x^4 + 8) \div (x - 3)$ using synthetic division. Write the division statement.

$$\begin{array}{r|rrrrr} -3 & 2 & 0 & 1 & -15 & 8 \\ & & -6 & -18 & -57 & -126 \\ \hline & 2 & 6 & 19 & 42 & 134 \end{array}$$

2 marks

$x^2 - 15x + 2x^4 + 8 = (x-3)(2x^3+6x^2+19x+42)+134$

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1 mark

3. Using the Remainder Theorem, determine the remainder when $x^3 + 7x^2 + 2x - 5$ is divided by $x + 7$.

$(-7)^3 + 7(-7)^2 + 2(-7) - 5 = -19$ ← zero = -7

The remainder is -19.

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4. What are possible factors of the polynomial function $f(x) = x^4 - 3x^3 - 8x^2 + 12x + 16$?

$x \pm 1,\ x \pm 2,\ x \pm 4,\ x \pm 8,\ x \pm 16$ ← factors of the constant

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5. Fully factor the polynomial function $f(x) = x^4 - 3x^3 - 8x^2 + 12x + 16$.

$f(1) = (1)^4 - 3(1)^3 - 8(1)^2 + 12(1) + 16 \neq 0$

$f(-1) = (-1)^4 - 3(-1)^3 - 8(-1)^2 + 12(-1) + 16 = 0 \rightarrow x+1$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -8 & 12 & 16 \\ & & 1 & -4 & -4 & 16 \\ \hline & 1 & -4 & -4 & 16 & 0 \end{array}$$

$\rightarrow (x+1)(x^3-4x^2-4x+16)$

↳ continue factoring

$f(2) = (2)^4 - 3(2)^3 - 8(2)^2 + 12(2) + 16 = 0 \rightarrow x-2$ is a factor

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -4 & 16 \\ & & -2 & 4 & 16 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$\rightarrow (x+1)(x-2)(x^2-2x-8)$

↳ factor

$f(x) = (x+1)(x-2)(x+2)(x-4)$

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zero = -3 remainder = 0

6. What value of a will ensure $x + 3$ is a factor of $ax^3 - 2x^2 + x - 6$? If the remainder is 7, what is a?

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$a(-3)^3 - 2(-3)^2 + (-3) - 6 = 0$

$-27a - 18 - 3 - 6 = 0$

$-27a - 27 = 0$

$-27a = 27$

$a = \frac{27}{-27} = -1$

$-27a - 18 - 3 - 6 = 7$

$-27a - 27 = 7$

$-27a = 34$

$a = -\frac{34}{27}$

7. State the following characteristics of the polynomial function $f(x) = -(x + 1)^3(x - 1)^2(x - 3)$ and then sketch the function.

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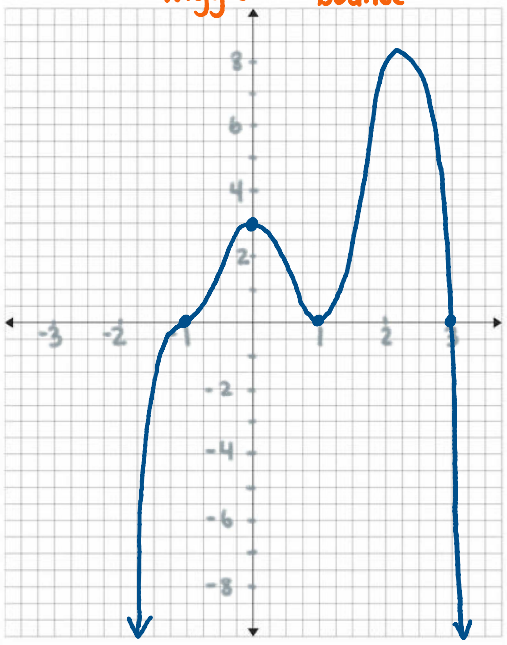
wiggle bounce

degree: 6 (add exponents)

sketch $f(x)$:



x-intercepts: $x = -1, 1, 3$

y-intercept: $f(0) = 3$

$-(0+1)^3(0-1)^2(0-3)$

sign of leading coefficient: negative

end behaviours: as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to -\infty$

8. Write a possible equation for a quintic function with zeros –5 and 4 of multiplicity 2, a zero 3 of multiplicity 1, and negative leading coefficient. Leave your answer in factored form.

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$f(x) = -(x+5)^2(x-4)^2(x-3)$

9. The height of a rectangular prism is 8 cm more than its width, and the depth is 3 cm less than its width. The volume is 1000 cm^3 more than the sum of its dimensions. Write an equation to determine the value of the width. Do not solve this problem!

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width: x

height: $x+8$

depth: $x-3$

sum $= x + x+8 + x-3 = 3x+5$

$x(x+8)(x-3) = 3x+5+1000$

$x(x+8)(x-3) = 3x+1005$

10. A piece of cardboard 20 cm wide and 30 cm long is used to make a box with no lid. Equal squares of side length x cm are cut from the corners and the sides are folded up. Write a polynomial function to represent the volume, V, of the box in terms of x and state the restrictions on x.

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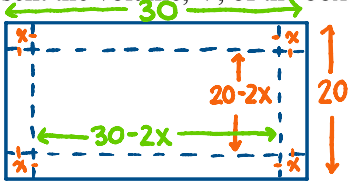

$V(x) = x(20-2x)(30-2x)$

$0 < x < 10$

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