PC12 Trig Identities Practice Test

Pre-Calc 12 **Trigonometric Identities Practice Test** Name:____________

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1. Write each expression in terms of a single trigonometric function. [3]

a) $\frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$

$= \cot^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

b) $1 - \sec^2\theta = -\tan^2\theta$

c) $\frac{2\tan\theta}{\tan^2\theta - 1} = \frac{2\tan\theta}{-(1-\tan^2\theta)}$

$= -\tan 2\theta$

2. State the non-permissible values of θ in each expression above. Answers should be in general form. [4]

p.609
p.612 #4

a) $\sin\theta \neq 0$

$\theta \neq 0, \pi, 2\pi, \ldots$

$\theta \neq \pi n, n \in \mathbb{Z}$

b) $\sec\theta = \frac{1}{\cos\theta}$

$\therefore \cos\theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\theta \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

or $\theta \neq \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

c) $\tan^2\theta \neq 1$

$\rightarrow \tan\theta \neq \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\rightarrow \cos\theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2} \ldots$

$\theta \neq \frac{(2n+1)\pi}{4}, \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

or $\theta \neq \frac{\pi}{4} + \frac{\pi}{2}n, \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

3. Write each expression in simplest form, then evaluate. [4]

a) $\sin\underset{\alpha}{\frac{\pi}{3}}\cos\underset{\beta}{\frac{\pi}{12}} - \cos\underset{\alpha}{\frac{\pi}{3}}\sin\underset{\beta}{\frac{\pi}{12}}$

$= \sin\left(\frac{\pi}{3} - \frac{\pi}{12}\right)$

$= \sin\left(\frac{4\pi}{12} - \frac{\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

b) $\left[\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)\right] + 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \cos\left(2\cdot\frac{\pi}{8}\right) + \sin\left(2\cdot\frac{\pi}{8}\right)$

$= \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)$

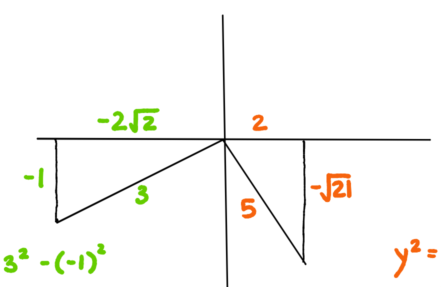
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{2}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

4. Given $\sin\beta = -\frac{1}{3}$ and $\cos\alpha = \frac{2}{5}$, where angle β is in standard position with its terminal arm in Q3 and angle α is in standard position with its terminal arm in Q4, determine the exact value of $\cos(\alpha + \beta)$. [4]



$x^2 = 3^2 - (-1)^2$

$= 9 - 1$

$= 8$

$x = -\sqrt{8} = -2\sqrt{2}$

$y^2 = 5^2 - 2^2$

$= 25 - 4$

$= 21$

$y = -\sqrt{21}$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{2}{5}\right)\left(\frac{-2\sqrt{2}}{3}\right) - \left(\frac{-\sqrt{21}}{5}\right)\left(-\frac{1}{3}\right)$

$= \frac{-4\sqrt{2}}{15} - \frac{\sqrt{21}}{15}$

$= \frac{-4\sqrt{2} - \sqrt{21}}{15}$

or $-\frac{4\sqrt{2} + \sqrt{21}}{15}$

p.637
p.644 #8
p.646 #13

5. Prove each identity. [15]

a) $\frac{\tan\theta\csc^2\theta}{\sec^2\theta} = \cot\theta$

$$LS = \frac{\tan\theta\csc^2\theta}{\sec^2\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta}}$$

$$= \frac{1}{\cos\theta\sin\theta} \div \frac{1}{\cos^2\theta}$$

$$= \frac{1}{\cancel{\cos}\theta\sin\theta} \times \frac{\cos^{\cancel{2}}\theta}{1}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

$$= RS$$

b) $\frac{\sin^2 x}{1+\cos x} = 1-\cos x$

$$LS = \frac{\sin^2 x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}$$

$$= \frac{\sin^2 x(1-\cos x)}{1-\cos^2 x}$$

$$= \frac{\cancel{\sin^2 x}(1-\cos x)}{\cancel{\sin^2 x}}$$

$$= 1-\cos x$$

$$= RS$$

or $LS = \frac{\sin^2 x}{1+\cos x}$

$$= \frac{1-\cos^2 x}{1+\cos x}$$

$$= \frac{\cancel{(1+\cos x)}(1-\cos x)}{\cancel{1+\cos x}}$$

$$= 1-\cos x$$

$$= RS$$

c) $\cot\theta = \frac{\sin 2\theta}{1-\cos 2\theta}$

$$RS = \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}$$

$1-1+2\sin^2\theta$

$1-1(1-2\sin^2\theta)$

$1-1+2\sin^2\theta$

$$= \frac{\cancel{2}\,\cancel{\sin}\theta\cos\theta}{\cancel{2}\sin^{\cancel{2}}\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

$$= LS$$

d) $\sin\left(\frac{\pi}{2}+x\right) = \cos x$

$$LS = \sin\left(\frac{\pi}{2}+x\right)$$

$$= \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x$$

$$= 1\cdot\cos x + 0\cdot\sin x$$

$$= \cos x$$

$$= RS$$

e) $\csc^2 x\sec^2 x = \csc^2 x + \sec^2 x$

$$RS = \csc^2 x + \sec^2 x$$

$$= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x\cos^2 x}$$

$$= \frac{1}{\sin^2 x\cos^2 x}$$

$$= \csc^2 x\sec^2 x$$

$$= LS$$