PC12 Trig Equations Practice Test

Pre-Calc 12 **Trigonometric Equations Practice Test** Name:_____

/30

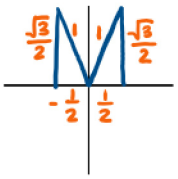
1. Solve the following equations for $0 \leq x < 2\pi$. Answers should be exact. Include a sketch indicating the appropriate quadrant(s) and special triangle applied in solving each equation. [6]

a) $2\sin x - \sqrt{3} = 0$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$



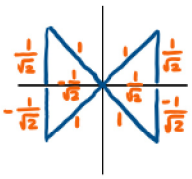
b) $2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



State the general solution for each equation in question 1. [2]

a) $x = \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, n \in \mathbb{Z}$

b) $x = \frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathbb{Z}$

2. Solve the following equations for $0 \leq x < 2\pi$. Answers should be exact. [8]

a) $2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ $\cos x = -1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $x = \pi$

$x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

b) $\sqrt{3}\cos x + \sin x = 0$

$\sqrt{3}\cos x = -\sin x$

$\sqrt{3} = -\frac{\sin x}{\cos x}$

$-\sqrt{3} = \tan x$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$

tangent ratios:

$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\tan\frac{\pi}{4} = 1$

$\tan\frac{\pi}{3} = \sqrt{3}$

c) $3\sin^2 x + \cos 2x - 2 = 0$

$3\sin^2 x + 1 - 2\sin^2 x - 2 = 0$

$\sin^2 x - 1 = 0$

$\sin^2 x = 1$

$\sin x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

d) $\sin 2\theta = \cos^2\theta$

$2\sin\theta\cos\theta - \cos^2\theta = 0$

$\cos\theta(2\sin\theta - \cos\theta) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin\theta = \cos\theta$

$2\cdot\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) \doteq 0.464, 3.605$

$\theta = 0.464, \frac{\pi}{2}, 3.605, \frac{3\pi}{2}$

expect answers to all be exact on the test sorry!

$\sin 2\theta = 2\cos^2\theta$

$\vdots$

$2\cos\theta(\sin\theta - \cos\theta) = 0$

$\sin\theta = \cos\theta$

$\tan\theta = 1$

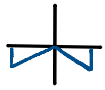
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

3. How many solutions are there in the interval $0 \leq x < 2\pi$ for the equation $\sin(bx) = -0.3$, where b is a positive integer? Provide a reason for your answer. [2]

There are 2b solutions since $\sin x = -0.3$ has two solutions per cycle and $\sin bx$ goes through b cycles in the interval $0 \leq x < 2\pi$.

4. Solve each equation over the domain $0 \leq x < 2\pi$. Then state the general solution. [6]

a) $\sin 2x = -\frac{1}{2}$



$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$+\pi$

b) $\tan 3x = \sqrt{3}$

$3x = \frac{\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}$

$+\frac{\pi}{3}$

general solution:

a) $x = \frac{7\pi}{12} + \pi n, \frac{11\pi}{12} + \pi n, n \in \mathbb{Z}$

b) $x = \frac{\pi}{9} + \frac{\pi}{3} n, n \in \mathbb{Z}$

* tangent only has one angle stated in the general solution *

5. Solve each equation over the domain $0 \leq x < 2\pi$. [6]

a) $\cos 4x \cos 2x + \sin 4x \sin 2x = -1$

$\cos(4x - 2x) = -1$

$\cos 2x = -1$

$2x = \pi, \ldots$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$+\pi$

b) $\sin 4x \cos x - \cos 4x \sin x = 0$

$\sin(4x - x) = 0$

$\sin 3x = 0$

$3x = 0, \pi, \ldots$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

$+\frac{2\pi}{3}$