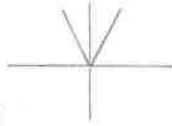


Solve each equation. Answers should be exact. Show all steps to receive full marks.

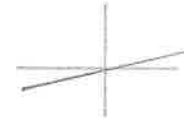
1. Solve each equation over the domain $0 \leq x \leq 2\pi$.

a) $\sin x = \frac{\sqrt{3}}{2}$



$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

b) $\tan x = \frac{1}{\sqrt{3}}$



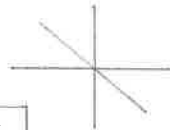
$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

2. Solve each equation over the domain $-\pi \leq x \leq \pi$.

a) $3 \tan x - 3 = 5 \tan x - 1$

$$-2 \tan x = 2$$

$$\tan x = -1$$



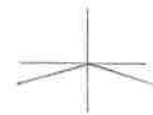
$$x = -\frac{\pi}{4}, \frac{3\pi}{4}$$

b) $5(1 + 2 \sin x) = 2 \sin x + 1$

$$5 + 10 \sin x = 2 \sin x + 1$$

$$8 \sin x = -4$$

$$\sin x = -\frac{1}{2}$$



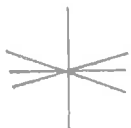
$$x = -\frac{\pi}{6}, -\frac{5\pi}{6}$$

3. Determine the general solution of each equation.

a) $4 \cos^2 x - 3 = 0$

$$4 \cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$



$$\cos x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \dots$$

$$x = \frac{\pi}{6} + \pi n, \frac{5\pi}{6} + \pi n, n \in \mathbb{Z}$$

b) $2 \cos^2 x - \cos x - 1 = 0$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\cos x = 1$$

$$x = 0, 2\pi, \dots$$

$$x = 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

2

2

4

8

4. Solve each equation over the domain $0 \leq x \leq 2\pi$.

a) $\cos x + \sqrt{3} \sin x = 0$

$$-\sqrt{3} \sin x = -\cos x$$

$$\frac{\sqrt{3} \sin x}{\cos x} = -1$$

$$\frac{\sin x}{\cos x} = \frac{-1}{\sqrt{3}}$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

b) $2 \cos x = 7 - 3 \sec x$ ← multiply each term by $\cos x$

$$2 \cos^2 x = 7 \cos x - 3$$

$$2 \cos^2 x - 7 \cos x + 3 = 0$$

$$(2 \cos x - 1)(\cos x - 3) = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = 3$$

(no values exist)

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

2

5. Solve each equation over the domain $0 \leq x \leq 2\pi$.

a) $\cos x + 1 = 2 \sin^2 x$

$$\cos x + 1 = 2(1 - \cos^2 x)$$

$$\cos x + 1 = 2 - 2 \cos^2 x$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \pi$$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

b) $\sqrt{2} \sin 2x - 2 \sin x = 0$

$$\sqrt{2} \cdot 2 \sin x \cos x - 2 \sin x = 0$$

$$2 \sin x (\sqrt{2} \cos x - 1) = 0$$

$$\sin x = 0$$

$$\cos x = \frac{1}{\sqrt{2}}$$

$$x = 0, \pi, 2\pi$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$x = 0, \frac{\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi$$

c) $3 \sin^2 x + \cos 2x - 2 = 0$

$$3 \sin^2 x + 1 - 2 \sin^2 x - 2 = 0$$

$$\sin^2 x - 1 = 0$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

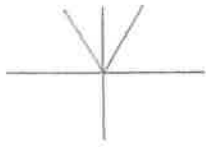
$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

6

Solve each equation. Answers should be exact. Show all steps to receive full marks.

1. Solve each equation over the domain $0 \leq x \leq 2\pi$. Then state the general solution.

a) $\sin 3x = \frac{\sqrt{3}}{2}$



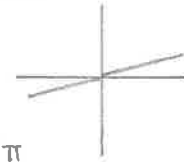
$$3x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$$

$$\underbrace{\hspace{10em}}_{+ \frac{2\pi}{3} \quad (6\pi/9) \quad + \frac{6\pi}{9}}$$

general solution: $x = \frac{\pi}{9} + \frac{2\pi}{3}n, \frac{2\pi}{9} + \frac{2\pi}{3}n, n \in \mathbb{Z}$

b) $\tan 2x = \frac{1}{\sqrt{3}}$



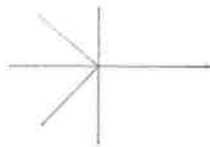
$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$$

$$\underbrace{\hspace{10em}}_{+ \frac{\pi}{2} \quad (6\pi/12)}$$

general solution: $x = \frac{\pi}{12} + \frac{\pi}{2}n, n \in \mathbb{Z}$

c) $\cos 2x = -\frac{1}{\sqrt{2}}$



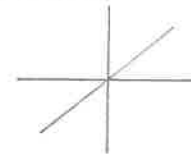
$$2x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$$

$$\underbrace{\hspace{10em}}_{+ \pi}$$

general solution: $x = \frac{3\pi}{8} + \pi n, \frac{5\pi}{8} + \pi n, n \in \mathbb{Z}$

d) $\tan 3x = 1$



$$3x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$$

$$\underbrace{\hspace{10em}}_{+ \frac{\pi}{3} \quad (4\pi/12)}$$

general solution: $x = \frac{\pi}{12} + \frac{\pi}{3}n, n \in \mathbb{Z}$

2. Solve each equation over the domain $0 \leq x \leq 2\pi$. Then state the general solution.

a) $\sin 7x \cos 5x - \cos 7x \sin 5x = -1$

$$\sin(7x - 5x) = -1$$

$$\sin 2x = -1$$

$$2x = \frac{3\pi}{2}$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\underbrace{\hspace{10em}}_{+ \pi}$$

general solution: $x = \frac{3\pi}{4} + \pi n, n \in \mathbb{Z}$

b) $\cos 2x \cos x - \sin 2x \sin x = 0$

$$\cos(2x + x) = 0$$

$$\cos 3x = 0$$

$$3x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$$

$$\underbrace{\hspace{10em}}_{+ \frac{2\pi}{3}}$$

general solution: $x = \frac{\pi}{6} + \frac{\pi}{3}n, n \in \mathbb{Z}$