8.3 Solving Systems by Substitution

Math 10   **Solving Systems of Linear Equations Algebraically**
                                     Substitution Method

Systems of equations can also be solved algebraically. This lesson looks at the substitution method, in which the expression of a variable is substituted into the other equation, and solved. Consider the following system:

$$① \quad 4x + 5y = 26$$
$$② \quad 3x = y - 9$$

The first step is to isolate one of the variables from one of the equations, whichever is easiest. Look for a variable which has a coefficient of 1 or −1.

$$② \quad 3x = y - 9$$
$$ +9 \qquad +9$$
$$3x + 9 = y$$

The second step is to substitute the expression for the isolated variable in the other equation. **If you have done this correctly, there will only be one variable in the equation.**

$$① \quad 4x + 5(3x + 9) = 26$$

The third step is to solve the equation.

$$4x + 5(3x + 9) = 26$$
$$4x + 15x + 45 = 26$$
$$19x + 45 = 26$$
$$ -45 \quad -45$$
$$19x = -19$$
$$\div 19 \quad \div 19$$
$$x = -1$$

The fourth step is to solve for the other variable.

→ use the simpler equation ← easier!
(either equation can be used)

$$3x = y - 9$$
$$3(-1) = y - 9$$
$$-3 = y - 9$$
$$+9 \qquad +9$$
$$6 = y$$

Once you have determined the value of each variable, state the solution.

$$(-1, 6) \quad \text{or} \quad x = -1 \text{ and } y = 6$$

It is helpful to check you answer to ensure it is correct. This is done by substituting the value of each variable into each equation and verifying the left side is equal to the right side.

① LS = 4x + 5y
= 4(-1) + 5(6)
= -4 + 30
= 26
= RS ✓

② LS = 3x   RS = y - 9
= 3(-1)   = 6 - 9
= -3   = -3
LS = RS
✓

Solve each system of linear equations by substitution. Verify your answer.

a) ① $2x + 5y = -18$
   ② $x + 2y = -6$

② $x = -2y - 6$

① $2(-2y-6) + 5y = -18$
   $-4y - 12 + 5y = -18$
   $y - 12 = -18$
   $+12 \quad +12$
   $y = -6$

$x + 2(-6) = -6$
$x - 12 = -6$
$+12 \quad +12$
$x = 6$

Solution: $(6, -6)$ ⤶ either
$x = 6$ and $y = -6$ ⤶

① $LS = 2x + 5y$
   $= 2(6) + 5(-6)$
   $= 12 - 30$
   $= -18$
   $= RS$ ✓

② $LS = x + 2y$
   $= 6 + 2(-6)$
   $= 6 - 12$
   $= -6$
   $= RS$ ✓

b) ① $3x + 5y = 27$
   ② $4x = 16$

② $\dfrac{4x}{4} = \dfrac{16}{4}$
   $x = 4$

① $3(4) + 5y = 27$
   $12 + 5y = 27$
   $-12 -12$
   $5y = 15$
   $\div 5 \quad \div 5$
   $y = 3$

Solution: $(4, 3)$
$x = 4$ and $y = 3$

check:

① $LS = 3x + 5y$
   $= 3(4) + 5(3)$
   $= 12 + 15$
   $= 27$
   $= RS$ ✓

② $LS = 4x$
   $= 4(4)$
   $= 16$
   $= RS$ ✓