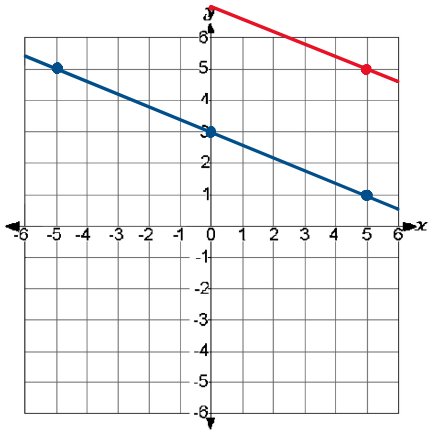
8.2 Number of Solutions of Linear Systems

Math 10 **Number of Solutions of Linear Systems**

When two lines are graphed on the same grid, they do not always have exactly one point of intersection. Consider the following systems of linear equations.

$4x + 10y = 30$
$2x + 5y = 35$

$10y = -4x + 30$
$y = -\frac{2}{5}x + 3$
$5y = -2x + 35$
$y = -\frac{2}{5}x + 7$

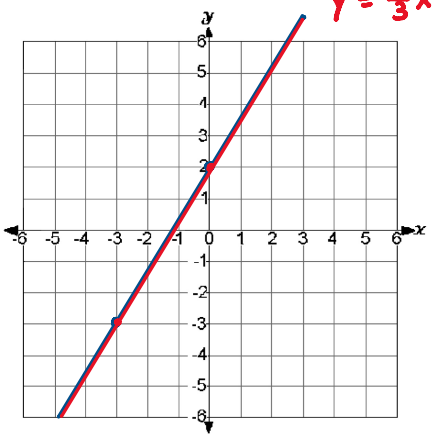


These lines are parallel.

This system has no solution.

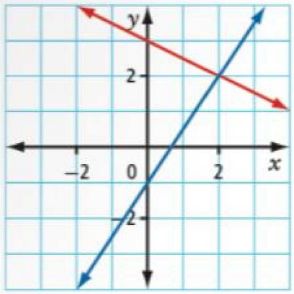
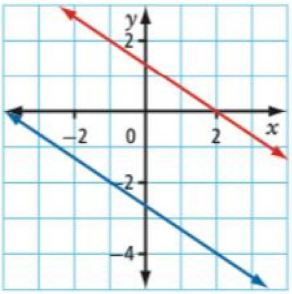
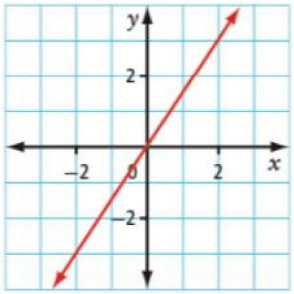
$10x - 6y = -12$
$21y = 42 + 35x$

$-6y = -10x - 12$
$y = \frac{5}{3}x + 2$
$y = 2 + \frac{35}{21}x$
$y = 2 + \frac{5}{3}x$
$y = \frac{5}{3}x + 2$
same!



These lines are coincidental.

This system has infinitely many solutions.

Intersecting Lines	Parallel Lines	Coincident Lines
one solution 	no solution 	an infinite number of solutions 
different slopes	same slope	same slope
y-intercepts can be the same or different.	different y-intercepts	same y-intercept

How many points of intersection does each of the following systems have?

a) $y = -\frac{1}{2}x + 4$
$x + 2y = 4$

$2y = -x + 4$
$y = -\frac{1}{2}x + 2$

→ slopes are equal
→ y-int are different
∴ parallel lines
∴ no solution

b) $y = \frac{2}{3}x + 1$
$6y - 4x = 6$

$6y = 4x + 6$
$y = \frac{2}{3}x + 1$

→ slopes are equal
→ y-int are equal
∴ coincidental lines
∴ infinitely many solutions

c) $y = 3x - 1$
$y = 2x - 1$

→ slopes are different
∴ one solution

It is possible to determine the number of solutions by comparing the coefficients of each equation, thus avoiding the tedious task of sketching lines on a grid, or converting each equation into slope-intercept form.

a) $2x + 3y = 12$
$2x + 3y = 20$

→ coefficients of x and y are the same in each equation
∴ lines are either parallel or coincidental
→ constants are different
∴ lines are not exactly the same
∴ parallel lines
∴ no solution

b) $10x + 15y = 60$ ①
$4x + 6y = 24$ ②

① divide each term by 5
$2x + 3y = 12$
② divide each term by 2
$2x + 3y = 12$
same equations!
∴ coincidental
∴ infinitely many solutions

c) $x + 2y + 4 = 0$ ①
$3x + 6y - 12 = 0$ ②

② divide each term by 3
$x + 2y - 4 = 0$
$x + 2y + 4 = 0$
→ coefficients of x and y are the same
→ constants are different (one is -4, the other is +4)
∴ parallel lines
∴ no solution

Assignment: handout