

horizontal expansion/compression and/or reflection

if $|k| > 1$: compression

if $|k| < 1$: expansion

if $k < 0$: reflection in y -axis

$$g(x) = a f [k (x - d)] + c$$

vertical shift

vertical expansion/compression
and/or reflection

if $|a| > 1$: expansion

if $|a| < 1$: compression

if $a < 0$: reflection in x -axis

horizontal shift
(opposite direction)

Quadratic Function:

$$g(x) = a [k(x-d)]^2 + c$$

Square Root Function:

$$g(x) = a \sqrt{k(x-d)} + c$$

Absolute Value Function:

$$g(x) = a | k(x-d) | + c$$

Does the order of transformations matter? Consider the following transformations.

a) Starting with $f(x) = |x|$, a reflection in the x -axis is followed by a horizontal stretch by a factor of 2, followed by a horizontal translation 3 units left, and by a vertical translation 4 units up. Determine the equation of the resulting function.

$$f(x) = |x| \rightarrow f(x) = -|x| \rightarrow f(x) = -|\frac{1}{2}x| \rightarrow f(x) = -|\frac{1}{2}(x+3)| + 4$$

b) Starting with $f(x) = |x|$, a reflection in the x -axis is followed by a horizontal translation 3 units left, followed by a horizontal stretch by a factor of 2, and by a vertical translation 4 units up. Determine the equation of the resulting function.

$$f(x) = |x| \rightarrow f(x) = -|x| \rightarrow f(x) = -|x+3| \rightarrow f(x) = -|\frac{1}{2}x+3| \\ \rightarrow f(x) = -|\frac{1}{2}x+3| + 4$$

When graphing functions, the order in which each transformation is applied matters!

For the function $f(x)$ shown on the graph below, apply the appropriate transformations to obtain a final function, $g(x) = -0.5f(x+2) + 3$. Then graph $g(x)$.

① ② ③ ④

Key Points $f(x)$	Transformations				Key Points $g(x)$
	-	0.5	2	3	
$(-4, 0)$	$\rightarrow (-4, 0)$	$\rightarrow (-4, 0)$	$\rightarrow (-6, 0)$	$\rightarrow (-6, 3)$	
$(-2, -2)$	$\rightarrow (-2, 2)$	$\rightarrow (-2, 1)$	$\rightarrow (-4, 1)$	$\rightarrow (-4, 4)$	
$(0, 0)$	$\rightarrow (0, 0)$	$\rightarrow (0, 0)$	$\rightarrow (-2, 0)$	$\rightarrow (-2, 3)$	
$(2, 4)$	$\rightarrow (2, -4)$	$\rightarrow (2, -2)$	$\rightarrow (0, -2)$	$\rightarrow (0, 1)$	
$(4, 0)$	$\rightarrow (4, 0)$	$\rightarrow (4, 0)$	$\rightarrow (2, 0)$	$\rightarrow (2, 3)$	

plot these points

