7.6 Example 4

Example 4 Using Double-Angle Identities to Solve an Equation

Solve the equation $\cos 2x = 1 - 2 \sin x$ over the domain $0 \leq x < 2\pi$.

SOLUTION

$\cos 2x = 1 - 2 \sin x$ Use the identity for $\cos 2x$ that involves $\sin x$.

$1 - 2 \sin^2 x = 1 - 2 \sin x$

$2 \sin^2 x - 2 \sin x = 0$ Factor.

$2 \sin x (\sin x - 1) = 0$

$2 \sin x = 0$ or $\sin x - 1 = 0$

$\sin x = 0$ $\qquad \sin x = 1$

$x = 0$ or $x = \pi$ $\qquad x = \frac{\pi}{2}$

Verify the solution.

The roots are: $x = 0$, $x = \pi$, and $x = \frac{\pi}{2}$

Check Your Understanding

4. Solve the equation
$\frac{1}{2} \sin 2x - \cos^2 x = 0$
over the domain $0 \leq x < 2\pi$.

$\frac{1}{2} \cdot 2 \sin x \cos x - \cos^2 x = 0$

$\sin x \cos x - \cos^2 x = 0$

$\cos x (\sin x - \cos x) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x - \cos x = 0$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\boxed{\therefore x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}}$

general solution:

$x = \frac{\pi}{4} + \pi n, \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

2π window

- $\tan x$ has 2 solutions
- $\sin x$, $\cos x$ have 2 solutions between -1 and 1
- $\sin x$, $\cos x$ have 1 solution for -1 or 1
- $\sin x$, $\cos x$ have no solutions for values less than -1 or greater than 1.

Discuss the Ideas

1. What is a double-angle identity? How is it different from double a trigonometric ratio?

2. Why are there three double-angle identities for $\cos 2\theta$ but only one identity for $\sin 2\theta$?

3. What are two identities for $\tan 2\theta$? Give an example of when it would be better to use each identity.