7.5 Example 4

Check Your Understanding

4. Solve the equation $\cos 4x \cos x + \sin 4x \sin x = 1$ over the domain $0 \leq x < 2\pi$.

$\cos(4x - x) = 1$

$\cos 3x = 1$

$3x = 0, 2\pi, 4\pi \ldots$

$\boxed{x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}}$

general solution:

$x = \frac{2\pi}{3}n, n \in \mathbb{Z}$

Example 4 Using the Sum and Difference Identities to Solve an Equation

Solve the equation $\sin 5x \cos 3x - \cos 5x \sin 3x = 1$ over the domain $0 \leq x < 2\pi$.

SOLUTION

$$\sin 5x \cos 3x - \cos 5x \sin 3x = 1$$
$$\sin(5x - 3x) = 1$$
$$\sin 2x = 1$$

The given domain for angle x is $0 \leq x < 2\pi$, so the domain for angle $2x$ is $0 \leq 2x < 4\pi$.

$2x = \frac{\pi}{2}$ or $2x = \frac{5\pi}{2}$

$x = \frac{\pi}{4}$ $\quad x = \frac{5\pi}{4}$

The roots are: $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

Discuss the Ideas

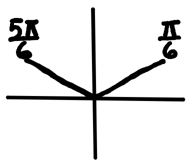
1. How do you know whether you can use a sum or difference identity to determine the exact value of a trigonometric ratio of a given angle?

2. What strategies do you have for remembering the sum and difference formulas?

More examples:

Solve each equation over the domain $0 \leq x \leq 2\pi$. Then state the general solution.

$\sin 3x = \frac{1}{2}$



period $= \frac{2\pi}{3}$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}$

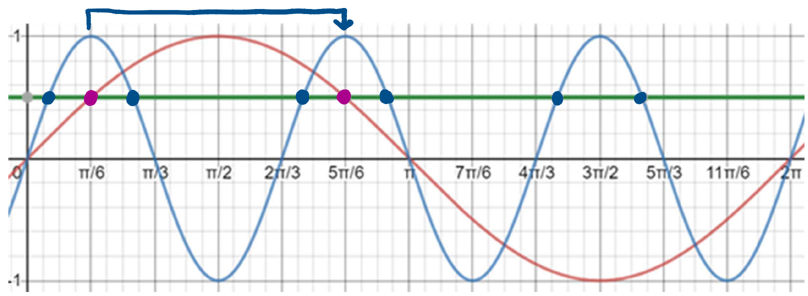
$x = \frac{\pi}{18}, \frac{5\pi}{18}$

$+\frac{2\pi}{3} \quad +\frac{2\pi}{3}$

$\frac{13\pi}{18}, \frac{17\pi}{18}$

$+\frac{2\pi}{3} \quad +\frac{2\pi}{3}$

$\frac{25\pi}{18}, \frac{29\pi}{18}$



$\frac{2\pi}{3} = \frac{12\pi}{18}$

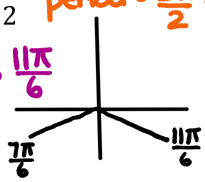
$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

general solution:

$x = \frac{\pi}{18} + \frac{2\pi}{3}n, \frac{5\pi}{18} + \frac{2\pi}{3}n, n \in \mathbb{Z}$

$\sin 2x = -\frac{1}{2}$ period $= \frac{2\pi}{2} = \pi$

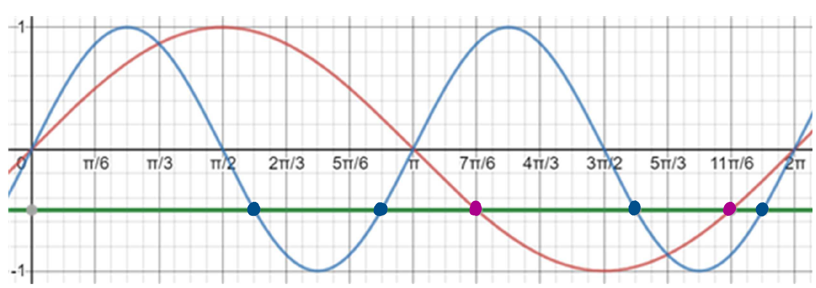
$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$



$x = \frac{7\pi}{12}, \frac{11\pi}{12}$

$+\pi \quad +\pi$

$\frac{19\pi}{12}, \frac{23\pi}{12}$



$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

general solution:

$x = \frac{7\pi}{12} + \pi n, \frac{11\pi}{12} + \pi n, n \in \mathbb{Z}$