

Solving Problems by Completing the Square

Example 1

The path of a volleyball can be modeled by the quadratic function $h(x) = -1.9x^2 + 7x + 0.3$ where $h(x)$ is the height of the volleyball and x is the horizontal distance the ball has travelled (both in metres).

- a) Find the starting height of the volleyball.

0.3m

- b) Find the maximum height of the volleyball. → Find the vertex

$$h(x) = -1.9(x^2 - 3.68x + 3.39) + 0.3 + 6.44$$

$$= -1.9(x - 1.84)^2 + 6.74$$

The maximum height is 6.74m.



$$\frac{-3.68}{2} = -1.84$$

$$(-1.84)^2 = 3.39$$

- c) How far (horizontally) does the ball travel before hitting the ground?

$$a = -1.9, b = 7, c = 0.3$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(-1.9)(0.3)}}{2(-1.9)}$$

$$= \frac{-7 \pm \sqrt{49 + 2.28}}{-3.8}$$

The ball travels 3.73m horizontally.

$$= -0.04, 3.73$$

- d) Suppose the net is exactly 3m from the location where the ball is hit. If the net is 2.1m tall, then will the ball clear the net?

$$-1.9(3)^2 + 7(3) + 0.3 = 4.2$$

Since the height at the net is greater than the net, it clears it.

Example 2

When bicycles are sold for \$300 each, a store can sell 70 in a season. For every \$25 increase in price it is estimated that the number of bicycles sold will decrease by 10. (or increase by 10 if the price decreases).

Find the price the store should sell the bikes for to maximize revenue. What is the maximum revenue?

$$\text{revenue} = \text{price} \times \# \text{ sold}$$

$$R = (300 + 25x)(70 - 10x)$$

Method 1: completing the square

$$R = (300 + 25x)(70 - 10x)$$

$$= 21000 - 3000x + 1750x - 250x^2$$

$$= -250x^2 - 1250x + 21000$$

$$= -250(x^2 + 5x + 6.25) + 21000 + 1562.5$$

$$= -250(x + 2.5)^2 + 22562.5$$

↑
max revenue

$$\text{new price} = 300 + 25(-2.5) = \underline{\underline{\$237.50}}$$

Method 2: find zeros

$$R = (300 + 25x)(70 - 10x)$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 300 + 25x = 0 \quad 70 - 10x = 0 \\ \frac{25x}{25} = \frac{-300}{25} \quad \frac{70}{10} = \frac{10x}{10} \\ x = -12 \quad 7 = x \end{array}$$

$$\text{midpoint: } \frac{-12 + 7}{2} = -2.5$$

$$\text{new price} = 300 + 25(-2.5) = \underline{\underline{\$237.50}}$$

$$\begin{aligned} \text{revenue} &= (237.50)(70 - 10(-2.5)) \\ &= \underline{\underline{\$22562.50}} \end{aligned}$$