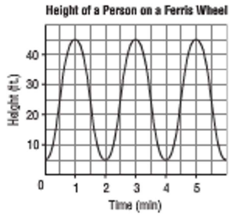
6.7 Applications

6.7 Applications of Sinusoidal Functions

FOCUS Model situations and solve problems using sinusoidal functions.

Get Started

When the last person gets on a Ferris wheel, it begins to rotate. This graph shows the height above the ground of that person, as a function of time.



How high above the ground does a person get on the Ferris wheel?

5 ft.

What is the radius of the Ferris wheel?

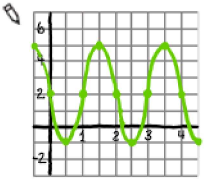
20 ft.

What is the time for one revolution of the wheel?

2 min.

Construct Understanding

Sketch the graph of $y = 3 \sin \pi(x - 1) + 2$ for $0 \leq x \leq 4$.
Describe your strategy.



amplitude: 3

equation of centre line: $y = 2$ $\quad -3 \leq y \leq 3$

$\quad +2 \quad +2$

→ range: $-1 \leq y \leq 5$

period: $\frac{2\pi}{\pi} = 2$

phase shift: 1

→ starting point: on the centre line at $x = 1$

Try: p.548 #3,4,6

In Lessons 6.4 to 6.6, the scale on the horizontal axis of a sinusoidal graph was in terms of π. When sinusoidal graphs are used in applications, the horizontal axis usually represents time, and the axis is labelled with whole numbers.

Phenomena, such as the oscillation of a mass on a spring or the height of a seat on a rotating Ferris wheel, produce measurements that vary between a maximum value and a minimum value. These phenomena can often be represented by a sinusoidal function. Since the graph of $y = \cos x$ has a maximum point on the y-axis, a cosine function may be used to model the data when the position of the first maximum or minimum is known. Then, the phase shift is the horizontal distance from the vertical axis to this point.

Check Your Understanding

1. A piston moves vertically in a cylinder starting from its minimum height. Every 20 s, the piston repeats its cycle from a minimum height of 15 cm to a maximum height of 35 cm back to a minimum height of 15 cm.

a) Determine a sinusoidal function that models the height, *h* centimetres, of the piston at time *t* seconds after it begins moving.

b) Use technology to graph the function, then estimate the height of the piston 26 s after it begins moving. Give the answer to the nearest centimetre.

min. height start

$\therefore h(t) = -\cos t$

period : 20

b: $\frac{2\pi}{20} = \frac{\pi}{10}$

amplitude :

a: $\frac{35-15}{2} = 10$

centre line :

d: $\frac{35+15}{2} = 25$

no phase shift since min value is starting point

$h(t) = -10\cos\frac{\pi}{10}t + 25$

Try #8

Example 1 Determining a Trigonometric Function that Models a Situation

The Singapore Flyer is the world's tallest Ferris wheel. People ride the wheel in capsules. The wheel has a diameter of 150 m and completes 1 revolution in approximately 32 min. A capsule reaches a height of 165 m.

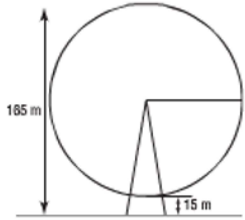
a) Determine a function that models the height, *h* metres, of a capsule at any time *t* minutes after the wheel begins to rotate.

b) Assume a capsule is at the base of the wheel when it begins to rotate. Use technology to graph the function, then estimate the height of the capsule 20 min after the wheel begins to rotate. Give the answer to the nearest metre.

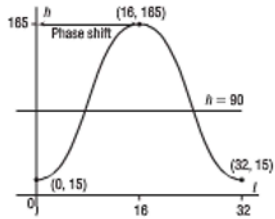
SOLUTION

a) Sketch a diagram.

A capsule travels along a circle at a constant speed, so its motion can be modelled using a sinusoidal function. The maximum height is 165 m and the diameter of the wheel is 150 m, so the minimum height is: 165 m − 150 m = 15 m



To sketch a graph:
the capsule is at the base of the wheel at time $t = 0$ and height $h = 15$.
So, the graph begins at (0, 15), which is a minimum point.
The maximum height is $h = 165$ after one-half of a revolution at $t = \frac{32}{2}$, or 16, so the first maximum point is at (16, 165).
The next minimum point is after 1 rotation and it has coordinates (32, 15).
The centre line of the graph has equation: $h = \frac{165 + 15}{2}$, or $h = 90$



The position of the first maximum is known, so use a cosine function to describe the motion: $h(t) = a\cos b(t - c) + d$

The amplitude is $165 - 90 = 75$, so $a = 75$.

The period is 32, so $b = \frac{2\pi}{32}$, or $\frac{\pi}{16}$.

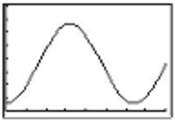
The phase shift is the *t*-coordinate of the first maximum point, so $c = 16$.

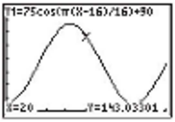
The vertical translation is $d = 90$.

So, an equation is: $h(t) = 75\cos\frac{\pi}{16}(t - 16) + 90$

$h(t) = -75\cos\frac{\pi}{16}t + 90$

b) Graph: $Y = 75 \cos \frac{\pi}{16}(X - 16) + 90$ (below left)
Determine the height after 20 min (below right).





After 20 min, the height of the capsule is approximately 143 m.

The solution to *Example 1b* can be determined algebraically.
To determine the height after 20 min, use:

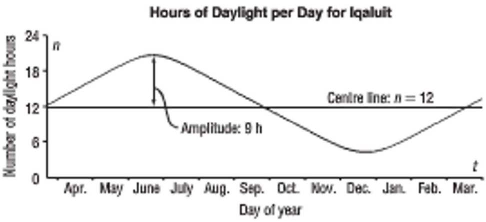
$h(t) = 75 \cos \frac{\pi}{16}(t - 16) + 90$ Substitute: $t = 20$

$h(20) = 75 \cos \frac{\pi}{16}(20 - 16) + 90$

$h(20) = 143.0330\ldots$

After 20 min, the height of the car is approximately 143 m.

Consider the graph from Lesson 6.5 *Get Started*, page 513. It shows the number of daylight hours, n, against day of the year, t, for Iqaluit. This graph approximates a sinusoidal function whose equation can be determined.



To write a function that models the data, the following assumptions are made:

- March 21 is day 0 on the *Day of year* axis. On this day, there are approximately 12 h of daylight. The maximum daylight is approximately 21 h on June 21.
- The function is sinusoidal with a period of approximately 365 days.

The graph approximates a sine curve with no phase shift, so it can be modelled with a function of the form $n(t) = a \sin bt + d$, where a is the amplitude, b is $\frac{2\pi}{\text{period}}$, and d is the vertical translation.

From the graph:

- The equation of the centre line is $n = 12$; so $d = 12$.
- The amplitude is 9 h; so $a = 9$.
- The period is 365 days; so $b = \frac{2\pi}{365}$

A possible function that models the number of hours of daylight in Iqaluit is: $n(t) = 9 \sin \frac{2\pi t}{365} + 12$

Example 2 Using a Sinusoidal Function to Model Given Data

The Fisheries and Oceans Canada website provides information about tide heights for many locations on the coastal regions of the country. The following data show the tide heights every 2 h for Digby, Nova Scotia, on March 9, 2011.

Time (hours after midnight)	00	02	04	06	08	10	12	14	16	18	20	22
Height (m)	6.3	7.8	6.1	3.2	1.4	2.7	5.6	7.5	6.3	3.7	1.7	2.6

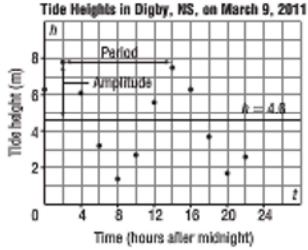
a) Graph the data, then write an equation of a sinusoidal function that models the data.

b) Use technology to graph the function in part a. Estimate the tide height at 16:15. Give the answer to the nearest tenth of a metre. How well does the function fit the data?

SOLUTION

a) Graph the data. Let h represent the height of the tide in metres, and t represent the time in hours after midnight.

Tide Heights in Digby, NS, on March 9, 2011



The beginning of the period is a maximum point, so use a cosine function to model the data: $h(t) = a \cos b(t - c) + d$

Check Your Understanding

2. The following data show the predicted tide heights every 2 h, starting at midnight, for St. Andrews, PEI, on March 9, 2011:
(00, 4.6), (02, 6.5), (04, 5.7), (06, 3.5), (08, 1.4), (10, 1.7), (12, 4.0), (14, 6.1), (16, 5.8), (18, 3.9), (20, 1.8), (22, 1.7)

a) Graph the data, then write an equation of a sinusoidal function that models the data.

b) Use technology to graph the function in part a. Estimate the tide height at 17:00. Give the answer to the nearest tenth of a metre.

period = time between maximum values

(02,6.5), (14,6.1) → 12 hours

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

amplitude = $\frac{\text{max}-\text{min}}{2} = \frac{6.5-1.4}{2} = 2.55$

$a \doteq 2.6$

centre line = $\frac{\text{max}+\text{min}}{2} = \frac{6.5+1.4}{2} = 3.95$

$d \doteq 4$

phase shift → first max (02,6.5)

$c = 2$

$\therefore h(t) = 2.6 \cos \frac{\pi}{6}(t-2) + 4$

Try # 1-6, 8, 9 (p.547)

From the graph:
The first maximum point has approximate coordinates (2, 7.8) and the first minimum point has approximate coordinates (8, 1.4),
so the equation of the centre line is approximately: $h = \frac{7.8 + 1.4}{2}$,
or $h = 4.6$; so $d \doteq 4.6$
The amplitude is approximately: $7.8 - 4.6 = 3.2$, so $a \doteq 3.2$ $\frac{7.8-1.4}{2} = 3.2$
The period is approximately 12 h, so $b \doteq \frac{2\pi}{12}$, or $\frac{\pi}{6}$
The phase shift is the t-coordinate of the first maximum point, so $c \doteq 2$
A function that approximates the data is: $h(t) = 3.2 \cos\frac{\pi}{6}(t - 2) + 4.6$

b) Graph: $Y = 3.2 \cos\frac{\pi}{6}(X - 2) + 4.6$
Write the time of 16:15 as a decimal of an hour: 16.25
Determine the height of the tide at this time.

At 16:15, the tide height is approximately 5.8 m.
The graph is not a perfect cosine function, but it is close enough to make a sinusoidal model reasonable.

Discuss the Ideas

1. Suppose you are to graph a sinusoidal function given its equation. How can you tell whether the horizontal axis will be labelled in terms of π or with integers?

2. How do you decide whether to use a sine function or a cosine function to model data that can be described with a sinusoidal function?