6.3 Infinite Geometric Series

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Compare the following geometric series:

$3 + 6 + 12 + 24 + \ldots 48$

This series diverges.

$S_1 = 3$

$S_2 = 9$

$S_3 = 21$

$S_4 = 45$

$S_5 = 93$

$3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \cdots \frac{3}{16}$

This series converges.

$S_1 = 3$

$S_2 = 4.5$

$S_3 = 5.25$

$S_4 = 5.625$

$S_5 = 5.8125$

If the sequence of partial sums **converges** to a constant value as the number of terms increases, then the geometric series is convergent and the constant value if the finite sum of the series. This is called the **sum to infinity** and is denoted by S_∞.

How can we determine whether the series converges?

If $-1 < r < 1$ the series converges.

$S_n = \frac{a(1-r^n)}{1-r}$ (r^n gets closer and closer to 0)

To determine the sum of an infinite series, use the formula:

$$S_\infty = \frac{a}{1-r}$$

Calculate the sum to infinity, if possible.

$32 + 8 + 2 + 0.5 + \cdots$ $a = 32;\ r = \frac{1}{4}$

$S_\infty = \frac{32}{1-1/4} = \frac{32}{3/4} = \frac{128}{3}$ or $42.\overline{6}$

$4 - \frac{4}{5} + \frac{4}{25} - \cdots$ $a = 4;\ r = -\frac{1}{5}$

$S_\infty = \frac{4}{1-(-1/5)} = \frac{4}{6/5} = \frac{20}{6}$ or $3.\overline{3}$

$4 - 6 + 9 - 13.5 + \ldots$ $a = 4;\ r = -3/2$ or -1.5

not possible

$\sum_{k=1}^{\infty} 100(-0.1)^{k-1}$ $a = 100;\ r = -0.1$

$S_\infty = \frac{100}{1-(-0.1)} = \frac{100}{1.1} = \frac{1000}{11}$ or $90.\overline{90}$

Determine a fraction that is equal to $0.1\overline{6}$.

$0.1\overline{6} = 0.1 + 0.06 + 0.006 + 0.0006 + \ldots$

$= 0.1 + \sum_{k=1}^{\infty} 0.06(0.1)^{k-1}$

$= \frac{1}{10} + \frac{0.06}{1-0.1}$ ← $\frac{a}{1-r}$

$= \frac{1}{10} + \frac{0.06}{0.9}$

$= \frac{1}{10} + \frac{6}{90}$

$= \frac{3}{} + \frac{2}{}$

$\therefore 0.1\overline{6} = \frac{1}{6}$

Assignment: handout

$= \frac{1}{10} + \frac{6}{90}$

$= \frac{3}{30} + \frac{2}{30}$

$= \frac{5}{30}$

$= \frac{1}{6}$

6.3 Assignment Name: ___________________

1. How do you determine whether an infinite geometric series diverges or converges?

2. Determine each sum, if possible.

a) $\sum_{k=1}^{\infty} 8\left(\frac{1}{4}\right)^{k-1}$

b) $-1 - \frac{3}{4} - \frac{9}{16} - \frac{27}{64} - \cdots$

c) $10 - \frac{20}{3} + \frac{40}{9} - \frac{80}{27} + \cdots$

d) $\sum_{k=1}^{\infty} -2\left(-\frac{1}{3}\right)^{k-1}$

3. Use the given data about each infinite geometric series to determine the indicated value.

a) $a = 21, S_\infty = 63$; determine r

b) $r = -\frac{3}{4}, S_\infty = \frac{24}{7}$; determine a

4. Use an infinite geometric series to express each repeating decimal as a fraction.

a) $0.4\overline{97}$

b) $1.\overline{143}$

5. Brad has a balance of $500 in a bank account. Each month he spends 40% of the balance remaining in the account.

a) Express the total amount Brad spends in the first 4 months as a series. Is the series geometric? Explain.

b) Determine the approximate amount Brad spends in 10 months.

c) Suppose Brad could continue this pattern of spending indefinitely. Would he eventually empty his bank account? Explain.

Answers:

2. a) $10.\overline{6}$ **b)** –4 **c)** 6 **d)** –1.5 **3. a)** $\frac{2}{3}$ **b)** 6 **4. a)** $\frac{493}{990}$ **b)** $\frac{1142}{999}$

5. a) $\$500(0.4) + \$500(0.6)(0.4) + \$500(0.6)^2(0.4) + \$500(0.6)^3(0.4)$; geometric **b)** $496.98