6.2 Arithmetic Series

Lesson 2: Arithmetic Series

An arithmetic series is the sum of the terms of an arithmetic sequence. For example, an arithmetic sequence is 5, 8, 11, 14, … The related arithmetic series is 5 + 8 + 11 + 14 + …

Find the sum of the first 5 terms of the arithmetic series $12 + 5 - 2 - \ldots$

$12 + 5 - 2 - 9 - 16 = -10$

The sum of the first n terms of an arithmetic series, S_n, can be determined using either of the following formulae:

$$S_n = \frac{n}{2}(t_1 + t_n) \qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

where t_1 is the first term, n is the number of terms, d is the common difference, and t_n is the nth term.

Example 1: Determine the sum of the arithmetic series $5 + 8 + 11 + \ldots + 53$.

① Determine which term is 53.

$t_n = 5 + (n-1)(3)$

$53 = 5 + (n-1)(3)$

$48 = (n-1)(3)$

$16 = n - 1 \rightarrow n = 17$

② Determine the sum.

$S_{17} = \frac{17}{2}(5 + 53)$

$= \frac{17}{2} \cdot 58$

$S_{17} = 493$

Example 2: Determine the sum of the first 10 terms of the arithmetic series:

a) $8 + 3 - 2 - \ldots$

$S_{10} = \frac{10}{2}[2(8) + (10-1)(-5)]$

$= 5[16 + -45]$

$= 5[-29]$

$= -145$

b) $\frac{2}{3} + \frac{5}{3} + \frac{8}{3} + \cdots$

$S_{10} = \frac{10}{2}[2(\frac{2}{3}) + 9(1)]$

$= 5[\frac{4}{3} + 9]$

$= 5[\frac{4}{3} + \frac{27}{3}]$

$= 5[\frac{31}{3}]$

$= \frac{155}{3}$ or $51\frac{2}{3}$ or $51.\overline{6}$

Example 3: Determine the sum, S_n, for each arithmetic series described.

a) $t_1 = 7$, $t_8 = 79$, $n = 8$

$S_8 = \frac{8}{2}(7 + 79)$

$= 4(86)$

$= 344$

b) $t_1 = 12$, $d = 8$, $n = 9$

$S_9 = \frac{9}{2}[2(12) + (9-1)(8)]$

$= \frac{9}{2}[24 + 64]$

$= \frac{9}{2}[88]$

$= 396$

Example 4: Determine the value of the first term if the sum of the first 18 terms is 279 and the common difference is –3.

$n = 18$
$S_{18} = 279$
$d = -3$

$S_n = \frac{n}{2}[2t_1 + (n-1)(d)]$
$279 = \frac{18}{2}[2t_1 + (18-1)(-3)]$
$279 = 9[2t_1 - 51]$
$\div 9 \quad \div 9$
$31 = 2t_1 - 51$
$+51 \quad +51$
$82 = 2t_1$
$\div 2 \quad \div 2$
$t_1 = 41$

Example 5: For the arithmetic series, determine the value of n if $t_1 = -6$, $t_n = 21$, and $S_n = 75$.

$S_n = \frac{n}{2}(t_1 + t_n)$
$75 = \frac{n}{2}(-6 + 21)$
$2 \cdot 75 = \frac{n}{2}(15) \cdot 2$
$150 = n(15)$
$\div 15 \quad \div 15$
$10 = n$

Example 6: Find t_{10} and S_{10} for the series $10 + 7 + 4 + \ldots$

$t_{10} = 10 + 9(-3)$
$= 10 - 27$
$= -17$

$S_{10} = \frac{10}{2}(10 - 17)$
$= 5(-7)$
$= -35$

Assignment **Arithmetic Series** Name: _____

1. Determine the sum of the arithmetic series $8 + 3 - 2 - \ldots -102$.

2. Determine the sum of the first 9 terms of the arithmetic series with $t_1 = 12$ and $d = 8$.

3. Determine the sum of the first 20 terms of the arithmetic series with $t_1 = -4$ and $t_{20} = 17$.

4. For the arithmetic series, determine the value of n if $t_1 = 8$, $t_n = 68$, and $S_n = 608$.

5. Find t_{10} and S_{10} for the series $-10 - 14 - 18 - \ldots$

6. A training program requires a pilot to fly circuits of an airfield. Each day, the pilot flies three more circuits that the previous day. On the fifth day, the pilot flies 14 circuits. How many circuits does the pilot fly

a) on the first day?

b) in total by the end of the 5^{th} day?

c) in total by the end of the n^{th} day?