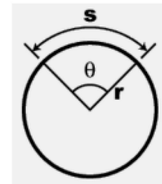
6.1 Radian Measure and Coterminal Angles

PC 12 **Radian Measure**

A convenient way to measure some angles, such as central angles, is radian measure: the ratio of the arc length (s) to the radius of a circle (r). Since each length is measured in the same unit, the radian has no units.



On a circle of radius r, the radian measure of a central angle θ that intersects an arc of length a is given by $\theta = \frac{s}{r}$.

1 radian is defined as the angle subtended by an arc length, s, equal to the radius, r since $\theta = \frac{r}{r} = 1$.

What is the arc length of a 360° angle? To answer this, we must know the circumference of a circle.

→ Circumference of a circle: $2\pi r$

→ The corresponding angle, θ, in radians is: $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$

∴ 360° = 2π radians or 180° = π radians

1 radian = $\frac{180°}{\pi}$ and $1° = \frac{\pi}{180}$ radians

Ex 1. a) Convert 60° to radians.

$$\cancel{60}^{1} \times \frac{\pi}{\cancel{180}_{3}} = \frac{\pi}{3}$$

b) Convert 150° to radians.

$$\cancel{150}^{5} \times \frac{\pi}{\cancel{180}_{6}} = \frac{5\pi}{6}$$

*Angles measured in radians are normally expressed without any units.

Ex 2. a) Convert $\frac{5\pi}{4}$ to degrees.

$$\frac{5\cancel{\pi}}{\cancel{4}_{1}} \times \frac{\cancel{180}^{45}}{\cancel{\pi}} = 225°$$

b) Convert $\frac{2\pi}{3}$ to degrees.

$$\frac{2\cancel{\pi}}{\cancel{3}_{1}} \times \frac{\cancel{180}^{60}}{\cancel{\pi}} = 120°$$

Ex 3. Determine the length of the arc in the diagram a) to the nearest tenth, and the measure of the angle in diagram b), in radians.

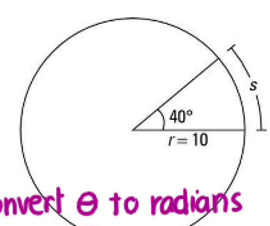
$\theta = \frac{s}{r}$

$r\theta = s$

$r = \frac{s}{\theta}$

*** θ must be in radians**

a)



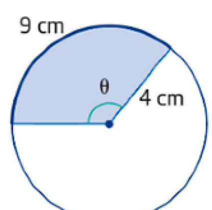
① convert θ to radians

$$4\cancel{0} \times \frac{\pi}{18\cancel{0}} = \frac{2\pi}{9}$$

② $s = r\theta = 10 \times \frac{2\pi}{9} = \frac{20\pi}{9}$

(≈ 7.0 units)

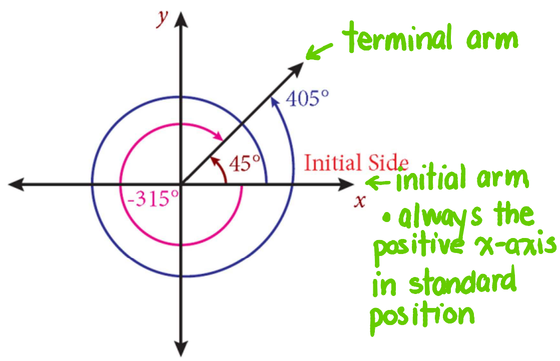
b)

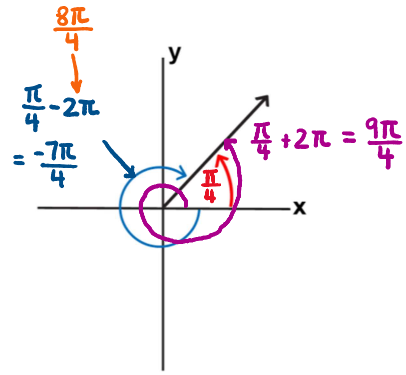


$\theta = \frac{s}{r} = \frac{9}{4}$ $(= 2.25)$

Coterminal Angles

Angles with the same terminal arm in standard position are called coterminal angles. For example, when you sketch –260°, 100° and 460° in standard position, they look the same.





What must be added to (or subtracted from) and angle, given in degrees, to determine the value of a coterminal angle? $360°$

What must be added to (or subtracted from) and angle, given in radians, to determine the value of a coterminal angle? 2π

Coterminal angles in general form:

Degrees: $\theta \pm 360n$	Radians: $\theta \pm 2\pi n$	*n is an integer

For each angle in standard position, determine one positive and one negative angle measure that is coterminal with it.

a) 270°

$270 + 360 = 630°$

$270 - 360 = -90°$

b) $-\frac{5\pi}{4}$

$-\frac{5\pi}{4} + \underset{\frac{8\pi}{4}}{2\pi} = \frac{3\pi}{4}$

$-\frac{5\pi}{4} - 2\pi = -\frac{13\pi}{4}$

c) 740°

$740 - \underset{720}{360(2)} = 20°$

$20 - 360 = -340°$

Express the angles coterminal with the angles above in general form.

a) $270 \pm 360n$ b) $-\frac{5\pi}{4} \pm 2\pi n$ c) $740 \pm 360n$

Assignment: #3, 4, 7, 9, 12 – 14 (handout)