6.1 Geometric Sequences

Pre-Calc 12 **Sequences and Series** Name: _____

Warm-up: Continue the pattern…

$3, 6, 9, 12, \underline{15}, \underline{18}, \ldots$ (add 3)

$25, 21, 17, 13, \underline{9}, \underline{5}, \ldots$ (subtract 4)

$1, 4, 9, 16, \underline{25}, \underline{36}, \ldots$ (squares)

$11, 10, 8, 5, \underline{1}, \underline{-4}, \ldots$ (subtract 1,2,3,...)

$3, 6, 12, 24, \underline{48}, \underline{96}, \ldots$ (multiply by 2)

$2, -6, 18, -54, \underline{162}, \underline{-486}$ (multiply by -3)

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \underline{\frac{1}{16}}, \underline{\frac{1}{32}}, \ldots$ (divide by 2 or multiply by $\frac{1}{2}$)

$1, 1, 2, 3, 5, 8, \underline{13}, \underline{21}, \ldots$ (Fibonacci sequence)

6.1 Geometric Sequences

A geometric sequence has a common ratio, r, which is multiplied by a term to generate the next term. Determine the value of r in each of these geometric sequences:

$2, 10, 50, 250, \ldots$ $r = 5$

$12, 6, 3, 1.5, 0.75, \ldots$ $r = \frac{1}{2}$

$3, -12, 48, -192, \ldots$ $r = -4$

$a, ab, ab^2, ab^3, \ldots$ $r = b$

Write the first four terms given the first term is -4 and the common ratio is 3.

$-4, -12, -36, -108$ (×3, ×3)

Determine the tenth term (t_{10}) and the general term (t_n) in the geometric sequence 3, 6, 12, …

$3, 6, 12, \underline{24}, \underline{48}, \underline{96}, \underline{192}, \underline{384}, \underline{768}, \underline{1536}, \ldots$ $t_n = 3 \cdot 2^{n-1}$

$3 \cdot 2^2 = 3 \cdot 2 \cdot 2$, $3 \cdot 2^4 = 3 \cdot 2 \cdot 2 \cdot 2$, $3 \cdot 2^5$ (term 6), $3 \cdot 2^7$, $3 \cdot 2^9$ (term 10)

1st term: $t_1 = a$
2nd term: $t_2 = ar$
3rd term: $t_3 = ar^2$
.
.
.

General term (n^{th} term) :

$$t_n = ar^{n-1}$$

Example 1: Using the formula for finding the general term, find t_{14} in the sequence 3, 6, 12, …

$a = 3$
$r = 2$
$n = 14$
$t_n = t_{14} = ?$

$t_{14} = 3 \cdot 2^{14-1}$
$= 3 \cdot 2^{13}$
$= 24576$

Which term has a value of 384?

$a = 3$
$r = 2$
$n = ?$
$t_n = 384$

$\frac{384}{3} = \frac{3 \cdot 2^{n-1}}{3}$
$128 = 2^{n-1}$
$2^7 = 2^{n-1}$
$\therefore 7 = n-1 \rightarrow 8 = n$

$\therefore$ The eighth term is 384.

Example 2: Find the eleventh term of a sequence if $t_4 = 3$ and $t_8 = 243$.

Method 1:

$t_4 = ar^3 = 3$; $t_8 = ar^7 = 243$

$\frac{ar^7}{ar^3} = \frac{243}{3}$

$r^4 = 81$
$r = \pm\sqrt[4]{81}$
$r = \pm 3$

$t_{11} = t_8 \cdot r^3$
$= 243 \cdot 3^3$ or $243(-3)^3$
$= 6561$ $= -6561$

Method 2:

reassign term values:
$k_1 = 3$, $k_5 = 243$ … find k_8

$k_5 = ar^4 = 243$

$\frac{3r^4}{3} = \frac{243}{3}$
$r^4 = 81$
$r = \pm 3$

$k_8 = k_5 \cdot r^3$
$= 243 \cdot 3^3$ or $243(-3)^3$
$= 6561$ $= -6561$

$\therefore t_{11} = 6561$ or -6561

Example 3: If $t_3 = 10$ and $t_6 = 80$, find t_8.

$\frac{ar^5}{ar^2} = \frac{80}{10}$
$r^3 = 8$
$r = 2$ * cannot be -2 *

$t_8 = 80 \cdot 2^2$
$= 320$

$k_1 = 10$, $k_4 = 80$ … find t_6

$10 \cdot r^3 = 80$
$r^3 = 8$
$r = 2$

$k_6 = 80 \cdot 2^2$ or $k_6 = 10 \cdot r^5$
$= 320$ $= 10 \cdot 2^5$
$= 320$

$\therefore t_8 = 320$

Assignment: handout

Assignment: handout

6.1 Assignment Name: _____

1. State the common ratio, then write the next 3 terms of each geometric sequence.

a) $-1, -3, -9, \ldots$

b) 48, 24, 12, …

c) $25, -50, 100, \ldots$

d) $4, -2, 1, \ldots$

e) $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \ldots$

f) $-24, 12, -6, \ldots$

2. For each geometric sequence, determine the indicated value.

a) 3, 6, 12, … ; determine t_7

b) 18, 9, 4.5, … ; determine t_6

c) 23, –46, 92, … ; determine t_{10}

d) $2, \frac{1}{2}, \frac{1}{8}, \ldots$; determine t_5

3. Write the first 4 terms of each geometric sequence, given the first term and the common ratio.

a) $t_1 = -3$; $r = 4$

b) $t_1 = 5$; $r = 2$

c) $t_1 = -0.5$; $r = -3$

d) $t_1 = \frac{1}{2}$; $r = \frac{2}{3}$

4. Use the given data about each finite geometric sequence to determine the indicated values.

a) Given $t_1 = -1$ and $r = -2$

i) Determine t_9.

ii) The last term is –4096. How many terms are in the sequence?

b) Given $t_1 = 0.002$ and $t_4 = 2$

i) Determine t_7.

ii) Determine which term has the value 20 000.

5. In a geometric sequence, $t_3 = 9$ and $t_6 = 1.125$; determine t_7 and t_9.

6. A beekeeper starts her business with 200 bees. New bees are hatched at a rate of 104% each week. How many bees were there after week 15?

7. A ball is dropped from a height of 25 m. After each bounce, the ball rises to 80% of the previous height.

a) Write the first 3 terms of a geometric sequence that models the height of the ball in metres.

b) To the nearest centimeter, to what height does the ball rise after the 5^{th} bounce?

c) To the nearest centimeter, to what height does the ball rise after the 10^{th} bounce?

d) After how many bounces does the ball rise to a height less than 1 m?

8. Between the Canadian censuses in 2001 and 2006, the number of people who could converse in Cree had increased by 7%. In 2006, 87 285 people could converse in Cree. Assume the 5–year increase continues to be 7%. Estimate to the nearest hundred how many people will be able to converse in Cree in 2031.

Answers:

1. a) 3; $-27, -81, -243$ **b)** 0.5; 6, 3, 1.5 **c)** $-2;\ -200{,}400, -800$ **d)** $-0.5;\ -0.5, 0.25, -0.125$ **e)** $\frac{1}{3}; \frac{1}{54}, \frac{1}{162}, \frac{1}{486}$ **f)** $-\frac{1}{2}; 3, -\frac{3}{2}, \frac{3}{4}$ **2. a)** 192 **b)** 0.5625 **c)** –11776 **d)** $\frac{1}{128}$ **3. a)** $-3, -12, -48, -192$ **b)** 5, 10, 20, 40 **c)** $-0.5, 1.5, -4.5, 13.5$ **d)** $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}$ **4. a) i)** –256 **ii)** 13 **b) i)** 2000 **ii)** t_8 **5.** 0.5625; 0.140625 **6.** 346 **7. a)** 20, 16, 12.8 **b)** 8.19 m **c)** 2.68 m **d)** 15 **8.** Approximately 122 400 people