

6.1 Arithmetic Sequences

Math 10

Arithmetic Sequences and Series

Name: _____

Warm-up: Continue the pattern...

3, 6, 9, 12, 15, 18, ... (add 3)

25, 21, 17, 13, 9, 5, ... (subtract 4)

1, 4, 9, 16, 25, 36, ... (add 3, 5, 7, 9, ... or squares)

11, 10, 8, 5, 1, -4, ... (subtract 1, 2, 3, ...)

3, 6, 12, 24, 48, 96, ... (multiplying by 2)

2, -6, 18, -54, 162, -486, ... (multiply by -3)

1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, ... (divide by 2 or multiply by $\frac{1}{2}$)

1, 1, 2, 3, 5, 8, 13, 21, ... (Fibonacci sequence)

Lesson 1: Arithmetic Sequences

An arithmetic sequence has a common difference, d , which is added to a term to generate the next term. Determine the value of d in each of these arithmetic sequences, then write the next three terms.

$2, 7, 12, 17, \dots$ 22, 27, 32

$$d = 5$$

$4, 5.5, 7, 8.5, \dots$ 10, 11.5, 13

$$d = 1.5$$

$3, -1, -5, -9, \dots$ -13, -17, -21

$$d = -4$$

$a, a - b, a - 2b, a - 3b, \dots$ $a - 4b$, $a - 5b$, $a - 6b$

$$d = -b$$

Write the first four terms given the first term is 4 and the common difference is -0.75 .

$4, 3.25, 2.5, 1.75$

Determine the tenth term (t_{10}) and the general term (t_n) in the arithmetic sequence 2, 5, 8, ...

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, ...

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2+3 & 2+3+3 & 2+5(3) & 2+9(3) & 2+12(3) & 2+15(3) \end{matrix}$

6th term 10th term

$$t_{10} = 2 + 9(3)$$

$$t_n = 2 + (n-1)(3)$$

1st term: $t_1 = t_1$
 2nd term: $t_2 = t_1 + d$
 3rd term: $t_3 = t_1 + 2d$

General term (n^{th} term):

$$t_n = t_1 + (n-1)d$$

Example 1: Using the formula for finding the general term, find t_{14} in the sequence $-1, 6, 13, \dots$

$$\begin{aligned} t_1 &= -1 & n &= 14 \\ d &= 7 & t_n &= ? \end{aligned} \qquad \begin{aligned} t_{14} &= -1 + (14-1)(7) \\ &= -1 + 13(7) \\ &= 90 \end{aligned}$$

Which term has a value of 167?

$$\begin{aligned} t_1 &= -1 & n &= ? \\ d &= 7 & t_n &= 167 \end{aligned} \qquad \begin{aligned} 167 &= -1 + (n-1)(7) \\ 168 &= (n-1)(7) \\ 24 &= n-1 \end{aligned} \qquad \begin{aligned} &\therefore \text{"therefore"} \\ &n = 25 \\ &\therefore t_{25} = 167 \end{aligned}$$

Example 2: A visual and performing arts group wants to hire a community events leader. The person will be paid \$22 for the first hour of work, \$37 for 2 h of work, \$52 for 3 h of work, and so on.

Write the general term that you could use to determine the pay for any number of hours worked.

$$t_n = 22 + (n-1)(15)$$

What will the person get paid for 6 h of work?

$$\begin{aligned} t_6 &= 22 + 5(15) \\ &= 22 + 75 \\ &= \$97 \end{aligned}$$

Example 3: The general guideline for the growth in height of a child between the ages of 3 years and 10 years is 5 cm per year. Suppose a child is 95 cm tall at age 3.

Write the general term that you could use to estimate what the child's height will be at any age between 3 and 10.

$$t_n = 95 + (n-1)(5) \qquad *n \text{ represent the number of years after age 3}$$

How tall is the child expected to be at age 10?

$$\begin{aligned} t_7 &= 95 + 6(5) \\ &= 125 \text{ cm} \end{aligned}$$

Example 4: A colony of ants increases by approximately 80 ants each month. Beginning with 40 ants, how many months would it take for the ant population to reach 3000?

$$\begin{aligned} 40 + (n-1)(80) &= 3000 \\ (n-1)(80) &= 2960 \\ n-1 &= 37 \\ n &= 38 \end{aligned} \qquad \text{It would take 38 months.}$$

Example 5: What is the charge for 10 h if a junior furnace technician charges \$45 for a house call plus \$46 per hour or portion of an hour?

$$\begin{aligned} t_n &= 45 + (n-1)(46) \\ t_{10} &= 45 + 9(46) \\ &= 459 \end{aligned} \qquad \text{The charge is \$459.}$$

Assignment**Arithmetic Sequences**

Name: _____

1. Decide whether each sequence is arithmetic. For each arithmetic sequence, state the value of t_1 , the value of d , and the next three terms.

a) 16, 32, 48, 64, 80, ____, ____, ____ $t_1 =$ _____ $d =$ _____

~~b) 2, 4, 8, 16, 32, ____, ____, ____ $t_1 =$ _____ $d =$ _____~~

c) -4, -7, -10, -13, -16, ____, ____, ____ $t_1 =$ _____ $d =$ _____

d) 3, 0, -3, -6, -9, ____, ____, ____ $t_1 =$ _____ $d =$ _____

2. Write the first four terms of each arithmetic sequence for the given values of t_1 and d .

a) $t_1 = 5, d = 3$ ____, ____, ____, ____ b) $t_1 = -1, d = -4$ ____, ____, ____, ____

c) $t_1 = 4, d = \frac{1}{5}$ ____, ____, ____, ____ b) $t_1 = 1.25, d = -0.25$ ____, ____, ____, ____

3. For each arithmetic sequence, determine the values of t_1 and d . State the missing terms of the sequence. Show how you determined the value of d .

a) ____, ____, ____, 19, 23 b) ____, ____, $3, \frac{3}{2}$ c) ____, 4, ____, ____, 10

4. Determine the position of the given term to complete each statement. Show your steps.

a) 170 is the ____th term of -4, 2, 8, ... b) -14 is the ____th term of $2\frac{1}{5}, 2, 1\frac{4}{5}, \dots$

c) 97 is the ____th term of -3, 1, 5, ... d) -10 is the ____th term of 14, 12.5, 11, ...

5. Determine the second and third terms of an arithmetic sequence if

a) $t_1 = 6$ and $t_4 = 33$

b) $t_1 = 8$ and $t_4 = 41$

c) $t_1 = 42$ and $t_4 = 27$

6. Determine the first term of the arithmetic sequence in which the 16th term is 110 and the common difference is 7.

7. the Big Sky Golf Club, located in Pemberton, was the site of the 2018 BC Senior Men's Championship. This tournament has a maximum entry of 156 players. The tee-off times begin at 8:00 am and are 8 min apart.

a) The tee-off times generate an arithmetic sequence. Write the first four terms of the arithmetic sequence if the first tee-off time of 8:00 am is considered to be at time 0.

b) Following his schedule, how many players will be on the course after 1 h, if the tee-off times are for groups of four?

c) Write the general term for the sequence of tee-off times.

d) At what time will the last group tee off?