

A paving company charges \$10/sq ft for installing interlocking paving stones, plus a \$50 delivery fee. The company calculates the cost to the customer as a function of the area to be paved. The owner of the company wants to express area in terms of cost to provide a quote of how much area can be paved for different budget amounts.

What is the cost function?

$$C(x) = 10x + 50$$

The **inverse of a function** is the reverse of the original function. What is the inverse of the cost function? What does it represent?

$$\begin{aligned} y &= 10x + 50 \\ x &= 10y + 50 \\ x - 50 &= 10y \\ \frac{x - 50}{10} &= y \end{aligned}$$

$$C^{-1}(x) = \frac{x - 50}{10}$$

Determining the Equation of a Function's Inverse

Example: Determine the inverse of each function below.

$$f(x) = 3x + 4$$

$$y = 3x + 4$$

$$x = 3y + 4 \quad \textcircled{1} \text{ switch } x \text{ \& } y$$

$$\begin{aligned} x - 4 &= 3y \\ \div 3 &\quad \div 3 \end{aligned} \quad \textcircled{2} \text{ isolate } y$$

$$\frac{x - 4}{3} = y \quad \textcircled{3} \text{ replace } y \text{ with } f^{-1}(x)$$

$$f^{-1}(x) = \frac{x - 4}{3}$$

$$g(x) = 8 - \frac{2}{5}x$$

$$y = 8 - \frac{2}{5}x$$

$$\begin{aligned} x &= 8 - \frac{2}{5}y \\ -8 &\quad -8 \end{aligned}$$

$$-\frac{5}{2} \cdot (x - 8) = -\frac{2}{5}y \cdot -\frac{5}{2} \quad \leftarrow \text{multiply by the reciprocal}$$

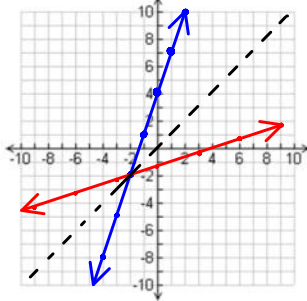
$$-\frac{5}{2}(x - 8) = y$$

$$g^{-1}(x) = -\frac{5}{2}(x - 8)$$

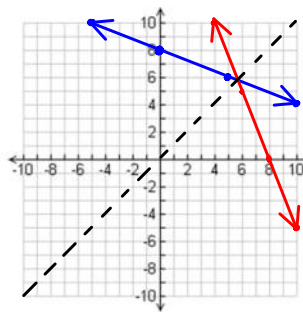
$$-\frac{5}{2} \cdot -8 = 20$$

How do these functions compare graphically?

slope  $\swarrow$   $\nwarrow$  y-int  
 $f(x) = 3x + 4$  "starting point"  
 $f^{-1}(x) = \frac{x}{3} - \frac{4}{3}$



$$g(x) = 8 - \frac{2}{5}x \quad g^{-1}(x) = -\frac{5}{2}x + 20$$



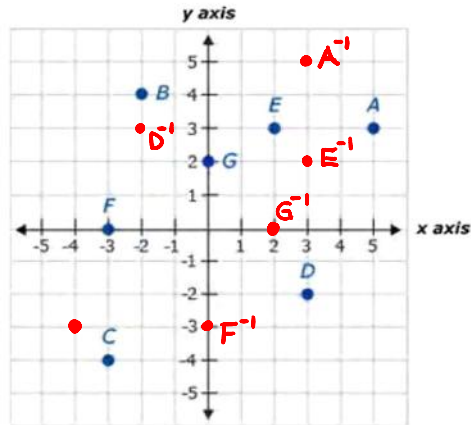
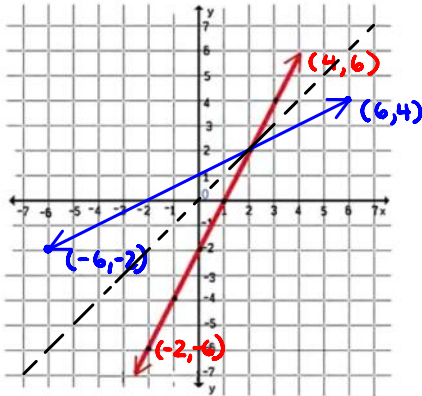
x	g^{-1}(x)
4	10
6	5
8	0
10	-5

↪ switch x & y

A function,  $h(x)$ , is represented by the set of points  $\{(-2, 6), (0, 8), (1, -5), (4, 8)\}$ . Determine  $h^{-1}(x)$ .

$$h^{-1}(x) = \{(6, -2), (8, 0), (-5, 1), (8, 4)\} \quad h(x): D: \{-2, 0, 1, 4\}$$
$$h^{-1}(x): D: \{-5, 6, 8\} \quad R: \{-5, 6, 8\}$$
$$R: \{-2, 0, 1, 4\}$$

Graph the inverse of each function.



How are the domain and range of a function and its inverse related?

The domain of a function is the range of its inverse.

The range of a function is the domain of its inverse.

(They are switched.)

Is the inverse of a function also a function?

Not always.  $h^{-1}(x)$  from above is not a function.