Completing the Square (Part 2)

Today we will look at completing the square for quadratic functions where the leading coefficient (a-value) is not equal to one.

As a warm-up, complete the square for the following:

$f(x) = x^2 - 6x + 1$

$= x^2 - 6x + 9 + 1 - 9$

$= (x-3)^2 - 8$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

We will work through an example of how to handle the case where $a \neq 1$.

$y = 2x^2 + 8x - 1$

$= 2(x^2 + 4x + 4\ ) - 1 - 8$

$= 2(x+2)^2 - 9$

$\frac{4}{2} = 2$

$2^2 = 4$

$f(x) = 3x^2 + 18x + 2$

$= 3(x^2 + 6x + 9\ ) + 2 - 27$

$= 3(x+3)^2 - 25$

$\frac{6}{2} = 3$

$3^2 = 9$

You try: $y = -2x^2 - 4x + 6$

$= -2(x^2 + 2x + 1\ ) + 6 + 2$

$= -2(x+1)^2 + 8$

Notice that fractions become necessary if the second coefficient is not divisible by a.

$y = 2x^2 + x - 1$

$= 2(x^2 + \frac{1}{2}x + \frac{1}{16}) - 1 - \frac{1}{8}$

$= 2(x + \frac{1}{4})^2 - 1\frac{1}{8}$

$\frac{1}{2} \div 2 = \frac{1}{4}$

$(\frac{1}{4})^2 = \frac{1}{16}$

$y = -2x^2 + 3x + 1$

You try... Complete the square for $f(x) = 3x^2 + 2x - 3$ and state the vertex.

It is also possible to complete the square in one step (if you derive the formula below.)

$y = ax^2 + bx + c$

Try it with...

$y = 2x^2 - 5x + 3$