5.6 Analyzing Logarithmic Functions

FOCUS Use technology to graph transformations of logarithmic functions.

Get Started

Use a calculator to evaluate each expression.

$\log 5$ $\qquad$ $2 \log 2$ $\qquad$ $\dfrac{\log 3}{\log 4}$

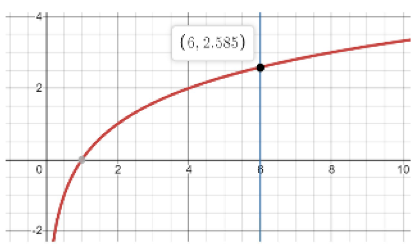
$\doteq 0.69897$ $\qquad$ $\doteq 0.60206$ $\qquad$ $\doteq 0.79248$

Construct Understanding

Evaluate $\log_2 6$.
Determine a strategy to verify your answer using graphing technology.

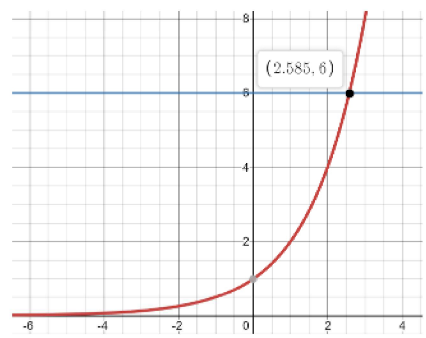
log → base 10
ln → base "e"

$y = \log_2 x$
$x = 6$



$\log_2 6 = x \Leftrightarrow 2^x = 6$

$y = 2^x$
$y = 6$



To use technology to evaluate a logarithm with base other than 10, the base of the logarithm has to be changed to 10.

Consider: $y = \log_b x$	Write this statement in exponential form.
$b^y = x$	Take the common logarithm of both sides.
$\log b^y = \log x$	Apply the power law to the left side.
$y \log b = \log x$	Divide both sides by $\log b$.
$y = \dfrac{\log x}{\log b}$	Substitute $\log_b x$ for y.

So, $\log_b x = \dfrac{\log x}{\log b}$

A similar relationship can be used to change the base b of a logarithm to any other base a.

Changing the Base of a Logarithm

Changing the Base of a Logarithm

$\log_b x = \dfrac{\log_a x}{\log_a b}$, where $a, b > 0; a, b \neq 1; x > 0$

Example 1 Using Technology to Approximate the Value of a Logarithm

Approximate the value of each logarithm, to the nearest thousandth. Write the related exponential expression.

a) $\log_6 100$ **b)** $\log_3 2$

SOLUTION

Use the change of base formula.

a) $\log_6 100 = \dfrac{\log 100}{\log 6}$
$= 2.5701\ldots$
$\log_6 100 \doteq 2.570$
So, $100 \doteq 6^{2.570}$

b) $\log_3 2 = \dfrac{\log 2}{\log 3}$
$= 0.6309\ldots$
$\log_3 2 \doteq 0.631$
So, $2 \doteq 3^{0.631}$

Check Your Understanding

1. Approximate the value of each logarithm, to the nearest thousandth. Write the related exponential expression.

a) $\log_5 50$ b) $\log_8 6$

a) $\log_5 50 = \dfrac{\log 50}{\log 5} \doteq 2.431$

So, $5^{2.431} \doteq 50$

b) $\log_8 6 = \dfrac{\log 6}{\log 8} \doteq 0.862$

So, $8^{0.862} \doteq 6$

THINK FURTHER

In *Example 1a*, explain why $\log_6 100 = \dfrac{2}{\log 6}$.

$\log_6 100 = \dfrac{\log 100}{\log 6} = \dfrac{\log 10^2}{\log 6} = \dfrac{2 \log 10}{\log 6} = \dfrac{2}{\log 6}$

1

Example 2 Using Technology to Graph a Logarithmic Function

a) Use a graphing calculator to graph $y = \log_4 x$.

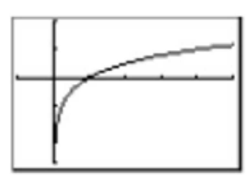
b) Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

SOLUTION

a) Use the change of base formula to change $y = \log_4 x$ to a logarithmic function with base 10.

$\log_4 x = \dfrac{\log x}{\log 4}$

Graph: $y = \dfrac{\log x}{\log 4}$



Check Your Understanding

2. a) Use a graphing calculator to graph $y = \log_5 x$.

b) Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

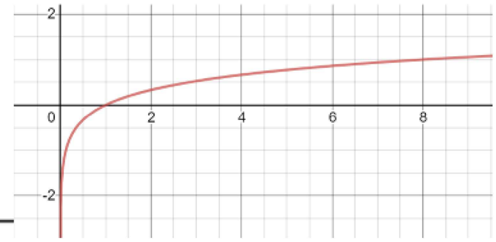
* Desmos allows bases other than 10 as well.

select functions
↳ Misc
↳ $\log_a$

b) Press [2nd] [TRACE] [2] for the zero feature from the CALC menu.

The x-intercept is 1.

b) Press $\boxed{2nd}$ $\boxed{TRACE}$ $\boxed{2}$ for the zero feature from the CALC menu.
The x-intercept is 1.
Since $\log x$ is not defined for $x \leq 0$, there is no y-intercept.
The equation of the asymptote is $x = 0$.
The domain of the function is $x > 0$.
The range of the function is $y \in \mathbb{R}$.



x-int: $x = 1$
y-int: none
asymptote: $x = 0$
domain: $x > 0$
range: $y \in \mathbb{R}$

THINK FURTHER

In *Example 2*, what other strategy could you use to verify the domain of the function?

Transformations can be applied to the graph of a logarithmic function.

Try #5, 7a (using Desmos) and 7b

The Function $y - k = c \log_a d(x - h)$, $a > 0$, $c \neq 0$, $d \neq 0$

When the graph of $y = \log_a x$ is:

- stretched vertically by a factor of $|c|$
- stretched horizontally by a factor of $\frac{1}{|d|}$
- reflected in the x-axis when $c < 0$
- reflected in the y-axis when $d < 0$
- translated k units vertically
- translated h units horizontally

the equation of the image graph is: $y - k = c \log_a d(x - h)$, $a > 0$

The general transformation is: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

Example 3 Transforming the Graph of a Logarithmic Function

a) Create a table of values for $y = \log_3 x$.
b) How is the graph of $y = \log_3(2x + 6)$ related to the graph of $y = \log_3 x$? Sketch these two graphs on the same grid.
c) Identify the intercepts and the equation of the asymptote of the graph of $y = \log_3(2x + 6)$, and the domain and range of the function.

Check Your Understanding

3. a) Create a table of values for $y = \log_2 x$.
b) How is the graph of $y = \log_2 2x - 1$ related to the graph of $y = \log_2 x$? Sketch these two graphs on the same grid.

c) Identify the intercepts and the equation of the asymptote of the graph of $y = \log_2 2x - 1$, and the domain and range of the function.

$y = \log_2 x \Leftrightarrow 2^y = x$

a)

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

b) The graphs of $y = \log_2 x$ and $y = \log_2 2x - 1$ are the same. Why?

$y = \log_2 2x - 1$

$y + 1 = \log_2 2x$

$y + 1 = \log_2 2 + \log_2 x$

$y + 1 = 1 + \log_2 x$

$-1 \quad -1$

$y = \log_2 x$

c) x-int: $x = 1$

y-int: none

asymptote: $x = 0$

domain: $x > 0$

range: $y \in \mathbb{R}$

Assignment: p. 405

#2, 5, 7ab, 8, 9ab*, 11

* list transformations & characteristics (do not graph)

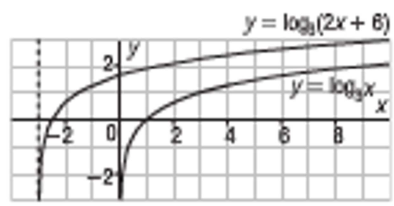
SOLUTION

a) To create the table of values, write $y = \log_3 x$ as $3^y = x$.

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

b) Write $y = \log_3(2x + 6)$ as $y = \log_3 2(x + 3)$.
Compare $y = \log_3 2(x + 3)$ with $y - k = c\log_3 d(x - h)$:
$k = 0, c = 1, d = 2$, and $h = -3$
The graph of $y = \log_3 2(x + 3)$ is the image of the graph of $y = \log_3 x$ after a horizontal compression by a factor of $\frac{1}{2}$, then a translation of 3 units left.
Use the general transformation:
(x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$
The point (x, y) on $y = \log_3 x$ corresponds to the point $\left(\frac{x}{2} - 3, y\right)$ on $y = \log_3 2(x + 3)$. Use the points (x, y) on $y = \log_3 x$.

(x, y)	$\left(\frac{x}{2} - 3, y\right)$
$\left(\frac{1}{9}, -2\right)$	$\left(-\frac{53}{18}, -2\right)$
$\left(\frac{1}{3}, -1\right)$	$\left(-\frac{17}{6}, -1\right)$
$(1, 0)$	$(-2.5, 0)$
$(3, 1)$	$(-1.5, 1)$
$(9, 2)$	$(1.5, 2)$



c) From the graph of $y = \log_3(2x + 6)$:
The x-intercept is -2.5.
For the y-intercept, substitute $x = 0$ in $y = \log_3(2x + 6)$.
$y = \log_3 6$
$y = \frac{\log 6}{\log 3}$
$y = 1.6309\ldots$
The y-intercept is approximately 1.6.
The equation of the asymptote is $x = -3$.
The domain of the function is $x > -3$.
The range of the function is $y \in \mathbb{R}$.

(do not graph)

The domain of the function is $x > -3$.
The range of the function is $y \in \mathbb{R}$.