5.3b Solving Exponential Equations Graphically

5.3 Solving Exponential Equations

FOCUS **Solve problems by modelling situations with exponential equations.**

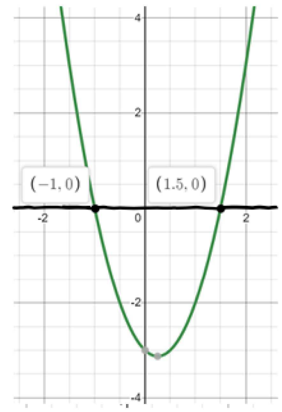
Get Started

Solve the equation $2x^2 - x - 3 = 0$ algebraically.
How could you solve it graphically?

$y = 2x^2 - x - 3$

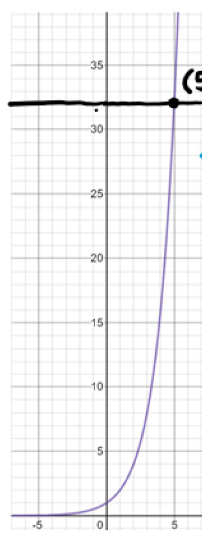
$y = 0$

$x = -1,\ 1.5$



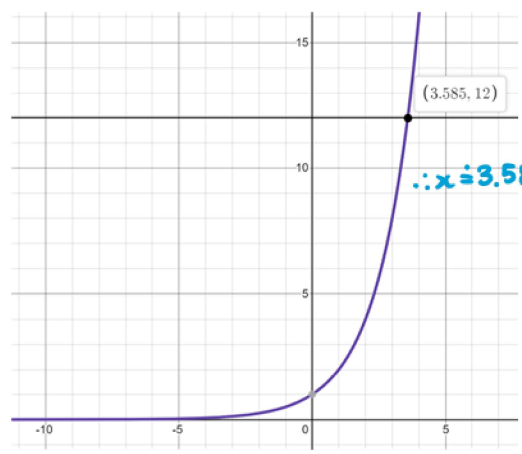
Construct Understanding

Solve this equation in two ways: $2^x = 32$
Can you use the same two strategies to solve this equation? $2^x = 12$
Explain your response.
Solve the equation and write the root to the nearest hundredth.



(5,32)

$\therefore x = 5$



$\therefore x \doteq 3.585$

THINK FURTHER

Is it possible for 2^x to be equal to any real number?

2^x can be equal to any real number greater than 0.

Example 3 Solving a Problem Involving Exponential Growth

A principal of \$1000 is invested at 6% annual interest, compounded monthly. To the nearest tenth of a year, when will the amount be \$1400?

SOLUTION

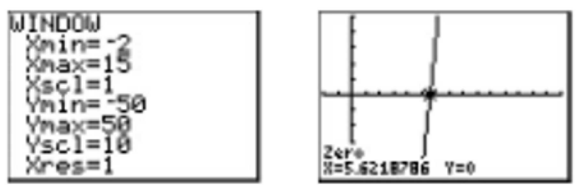
Use: $A = A_0\left(1 + \frac{i}{n}\right)^{nt}$, where t is the time in years since the principal was invested

Substitute: $A = 1400$, $A_0 = 1000$, $i = 0.06$, $n = 12$

$1400 = 1000\left(1 + \frac{0.06}{12}\right)^{12t}$

Since both sides of the equation cannot be written with the same base, use a graphing calculator to graph a related function.

Graph $y = 1000\left(1 + \frac{0.06}{12}\right)^{12x} - 1400$, then determine the approximate zero of the function: 5.6218786



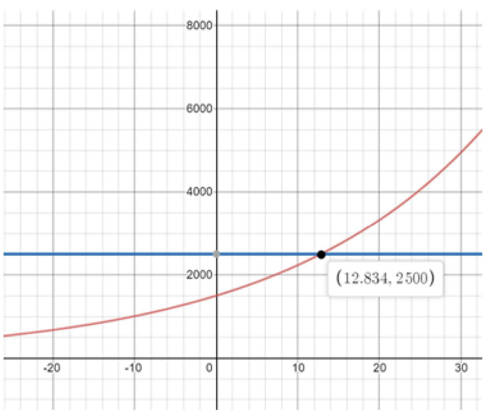
The amount will be \$1400 in approximately 5.6 years.

Check Your Understanding

3. A principal of \$1500 is invested at 4% annual interest, compounded quarterly. To the nearest quarter of a year, when will the amount be \$2500?

$y = 1500\left(1 + \frac{0.04}{4}\right)^{4x}$

$y = 2500$



It will take approximately 12.75 years.

$1 + \frac{0.04}{4} = 1.01 > 1$

$\therefore$ exponential growth

Atmospheric pressure decreases by about 12% for every 1 km increase in altitude. At sea level, atmospheric pressure is approximately 101.3 kilopascals (kPa).

So, at 1 km altitude, the pressure in kilopascals is:

88% of $101.3 = 101.3(0.88)$

At 2 km, the pressure in kilopascals is:

88% of $101.3(0.88) = 101.3(0.88)^2$

At 3 km, the pressure in kilopascals is:

88% of $101.3(0.88)^2 = 101.3(0.88)^3$

This pattern continues.

At an altitude of h kilometres, the pressure, P kilopascals, is modelled by the function: $P = 101.3(0.88)^h$

The function $P = 101.3(0.88)^h$ is an example of **exponential decay**.

Exponential Decay

A function that models exponential decay has the form: $y = ak^{bx}$
where $0 < k^b < 1$, and $a \in \mathbb{R}, b \in \mathbb{R}, k > 0$
k is the *decay factor*.

Example 4 Solving a Problem Involving Exponential Decay

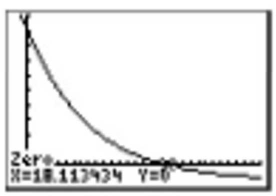
The function $P = 101.3(0.88)^h$ models the atmospheric pressure, P kilopascals, at an altitude of h kilometres. To the nearest kilometre, at what altitude is the atmospheric pressure only 10 kPa?

SOLUTION

Use: $P = 101.3(0.88)^h$ Substitute: $P = 10$

$10 = 101.3(0.88)^h$

Use a graphing calculator to graph a related function.
Graph $y = 101.3(0.88)^x - 10$, then determine the approximate zero of the function: 18.113434

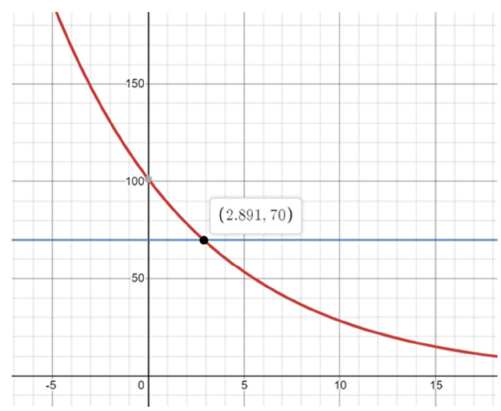


The atmospheric pressure is 10 kPa at approximately 18 km altitude.

Check Your Understanding

4. If the cabin pressure in an airplane is less than 70 kPa, passengers can suffer altitude sickness. To the nearest kilometre, at what altitude is the atmospheric pressure 70 kPa?

$y = 101.3(0.88)^x$
$y = 70$



$\therefore$ The atmospheric pressure is 70 kPa at an altitude of approximately 3 km.

Assignment:
p. 366 # 11ac, 12, 13a) i, iv
(use Desmos)

Discuss the Ideas

1. What is an exponential equation?

2. How do you identify whether a given function models exponential decay or exponential growth?