5.3 Domain and Range

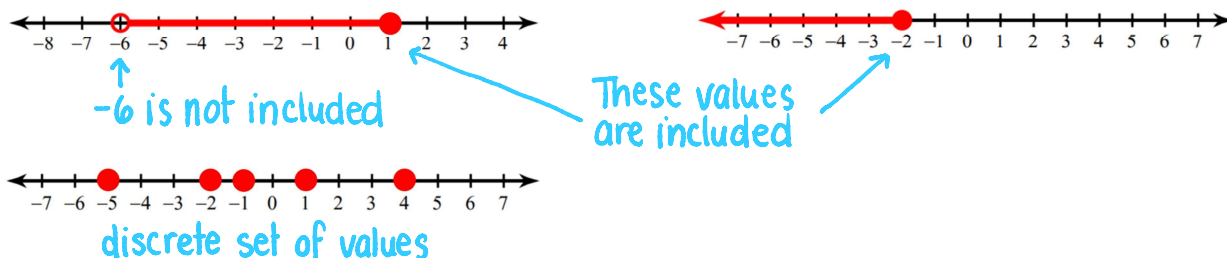**Domain and Range**

The domain and range describe the values that are included/possible in a relation. Specifically, the domain is the set of all possible values for the independent variable in a relation (all possible inputs). The range is the set of all possible values for the dependent variable (all possible outputs). There are a variety of ways we can express a set of numbers.

Number Lines:
If a set of numbers is continuous, the endpoints are joined by a solid line. If the endpoints are solid, they are included. If the endpoints are open, they are not included. If a set of numbers is discrete, the points are drawn on the number line without being joined.

−6 is not included

These values are included

discrete set of values

Lists:
Lists are useful when there aren't many numbers in the set. For the relation $(0, 0), (1, 5), (3, 7), (5, 7)$, the domain is $\{0, 1, 3, 5\}$ and the range is $\{0, 5, 7\}$.
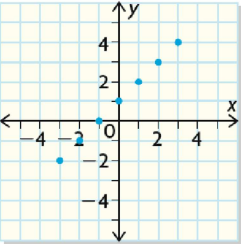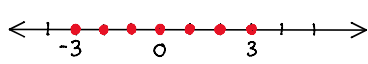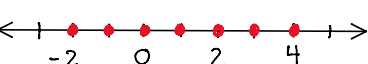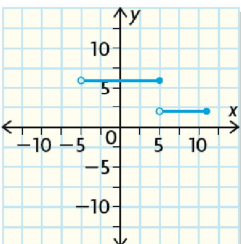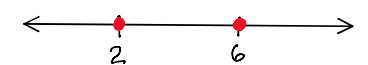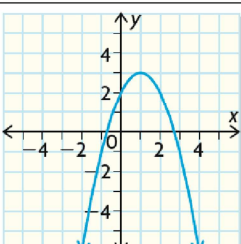
Set Notation: This is the formal mathematical way to give the values of the domain and range.

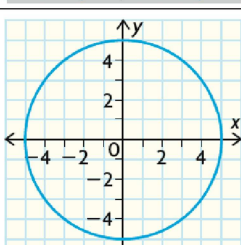| Set Notation	What It Means
The domain: $\{x \mid x \le 10, x \in \mathbb{R}\}$	$\{\}$ are the type of brackets used for a set rational #'s: $\mathbb{Q}$ whole #'s: $\mathbb{W}$ $\in$ means "is an element of" irrational #'s: $\overline{\mathbb{Q}}$ natural #'s: $\mathbb{N}$ $\mid$ means "such that" integers: $\mathbb{Z}$ "The domain is the set of values x such that x is less than or equal to 10, and x is an element of the real numbers."
The range: $\{y \mid y > 20, y \in \mathbb{R}\}$	"The set of values y such that y is greater than 20, and y is an element of the real numbers."

Interval Notation: This notation uses brackets to indicate an interval.

| Interval Notation	What It Means
The domain: $[0, 10]$	A straight bracket [or] is used if the end number is included. [0, 10] means "all numbers between 0 and 10, inclusive."
The range: $(20, \infty)$	A round bracket (or) is used if the end number is not included. The infinity symbol ∞ is used if there is no end point. $(20, \infty)$ means all numbers greater than 20

Example: Determine the domain and range for each graph.

	Number Line	Set and Interval Notation, or List
a) graph	D: number line, points at -3 to 3 R: number line, points -2 to 4	D: $\{-3,-2,-1,0,1,2,3\}$ R: $\{-2,-1,0,1,2,3,4\}$
b) graph	D: number line, open at -5, closed at 11 R: number line, points 2 and 6	D: $\{x \mid -5 < x \leq 11, x \in R\}$ $(-5, 11]$ R: $\{2, 6\}$
c) graph	D: full line R: line up to 3	D: $\{x \in R\}$ $(-\infty, \infty)$ R: $\{y \mid y \leq 3, y \in R\}$ $(-\infty, 3]$
d) graph	D: line from -5 to 5 R: line from -5 to 5	D: $\{x \mid -5 \leq x \leq 5, x \in R\}$ $[-5, 5]$ R: $\{y \mid -5 \leq y \leq 5, y \in R\}$ $[-5, 5]$

Example: A rectangular garden is enclosed using 24 m of fencing. Express the area of the garden in relation to its width.

A = w(w - 12)

w

12 - w
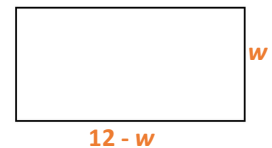
What are the domain and range of the area relation?

Domain = width   Range = area

D: {*w* | 0 < *w* < 12, *w* E R}

R: {*A* | 0 < *A* ≤ 36, *A* E R}

Assignment: p. 123 #1 – 4

Width (m)	Length (m)	Area (m²)
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36
6.5	5.5	35.75
0.5	11.5	5.75