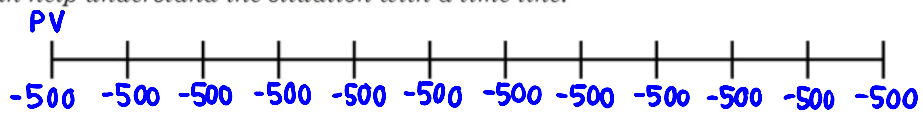
MCF3M **Lesson 4: Present Value of Annuities**

A student wants to have $500 every month for the first year of college to help pay for living expenses. They will keep the money in a savings account (paying 3%/a compounded monthly) and make regular withdrawals at the end of each month. What starting balance (present value) of the account is needed at the beginning of the year in order to have this money available?

We can help understand the situation with a time line.



This is an example of a totally different type of annuity. In this annuity there is a starting balance and a person withdraws money as the balance collects interest. At the end of the annuity the money is all gone.

There is a **present value formula** for this type of annuity:

present value → $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$ ← interest

R: payment/withdrawal value

n: number of payments/withdrawals

Use the formula to find out how much the student above needs to have saved.

$$\frac{500\left[1-\left(1+\frac{0.03}{12}\right)^{-12}\right]}{\frac{0.03}{12}} = 5904.63$$

The student needs to have saved $5904.63.

Example: Ms Kob wants to live off of $3500 a month when she retires. Her retirement income plan will pay 8%/a compounded monthly. If Ms Kob needs retirement income for 30 years after she retires, how much must she have saved by the time she retires?

$n = 30 \times 12 = 360$

$$\frac{3500\left[1-\left(1+\frac{0.08}{12}\right)^{-360}\right]}{\frac{0.08}{12}} = 476992.23$$

Ms Kob needs to have saved $476992.23.

Example: Suppose "Joe" retires with $450,000. Using the same amounts from the last example, what monthly payment amount could Joe receive from an annuity?

$$\frac{R\left[1-\left(1+\frac{0.08}{12}\right)^{-360}\right]}{\frac{0.08}{12}} = 450\,000$$

$$\frac{R(136.2834941)}{136.2834941} = \frac{450\,000}{136.2834941}$$

$$R = 3301.94$$

Joe could receive $3301.94 monthly.

Example: A family donates a lump sum of money to be given out as a math scholarship award to a graduating high school student for the next 20 years. The donation amount is $50,000. The money collects interest in an annuity at 9%/a compounded annually and is invested one year before the scholarship is given out. What amount can be given out as the scholarship reward each year?

$$\frac{R\left[1-(1+0.09)^{-20}\right]}{0.09} = 50000$$

$$\frac{R(9.128545669)}{9.128545669} = \frac{50000}{9.128545669}$$

$$R = 5477.32$$

$5477.32 can be given out each year.

Winning the Lottery

Suppose you win a five million dollar jackpot in the lottery. You decide to invest the money into an annuity paying 4%/a compounded weekly. What weekly payment could you withdraw that would allow the money to last for the next 75 years?

$n = 75 \times 52 = 3900$

$$\frac{R\left[1-\left(1+\frac{0.04}{52}\right)^{-3900}\right]}{\frac{0.04}{52}} = 5\,000\,000$$

$$\frac{R(1235.202126)}{1235.202126} = \frac{5\,000\,000}{1235.202126}$$

$$R = 4047.92$$

You could withdraw $4047.92 per week.