Exponential Growth & Decay

Last day we looked at several examples of exponential functions in the "real world".

Breeding Dogs: 3 initial dogs, number of dogs doubled each generation

Facebook Party: 1 initial invite, number of invites triples each day

McDonald's: $9 per hour, increase of 4% each year

Humpback Whales: 652 whales, increase of 6.5% each year

Car Depreciation: $42,000 car, decrease of 20% value each year

Detroit City: 950,000 people decrease of 1.5% each year

The above are examples of exponential growth or exponential decay.

There is a general formula for exponential growth/decay:

$$y = (a)b^x$$

Where a is always the initial value/population

Where b is always the rate of change per period x

y will then give you the population/amount at time period x.

if $b > 1$ → exponential growth
if $b = 1$ → no change
if $b < 1$ → exponential decay

Examples

1. The population of rabbits doubles every year in Mr. Elliott's back yard. This year Mr. Elliott counts 20 rabbits. How many rabbits will there be in

a) 3 years?

$y = 20(2)^x$

$y = 20(2)^3$

$= 20(8)$

$= 160$ rabbits

b) 10 years?

$y = 20(2)^{10}$

$= 20(1024)$

$= 20480$ rabbits

2. Suppose that a town has a population in the year 2010 of 37,000 people and is increasing by 2.1% each year.
   a) Define an equation to model the population of the town.

$P = 37000(1.021)^x$

$= 0.021$

next year's population is 102.1%

$102.1 \div 100 = 1.021$

   b) Use your equation to predict the population of the town in:
      i) 2019 ii) 2008

$P = 37000(1.021)^9$
$= 44610$

$P = 37000(1.021)^{-2}$
$= 35494$

3. A tire with a slow puncture loses pressure at a rate of 4% /min. If the tire's pressure is 300 kPa to begin with, predict the pressure of the tire after 17 minutes. $= 0.04$

$P = 300(0.96)^x$
$P = 300(0.96)^{17}$
$= 150$ kPa

$100 - 4\% = 96\%$
$1 - 0.04 = 0.96$

4. The population of Calgary is growing exponentially. The population is summarized in the table below:

Year	Population
2005	895,000
2006	949,781
2007	1,007,915

$949781 \div 89500 = 1.0612$
$1007915 \div 949781 = 1.0612$

Define an equation that models the population of Calgary and predict the population of Calgary in the year 2015.

$P = 895000(1.0612)^{10}$
$= 1621046$ people

Questions

1. The population of Alberta can be modelled by the equation $P = 2.238(1.014)^n$ where *P* is the population in millions and *n* is the number of years since 1981.

a) What was the population of Alberta in 1981?
b) At what annual rate, as a percent, has Alberta's population been increasing since 1981?
c) Estimate the population in 2021.

2. Ontario's population in 1991 was approximately 10.1 million. The population has been increasing at a rate of 1.25% per year.

a) Write an equation to represent the population of Ontario, *y* millions and the number of years, *x* since 1991.
b) Use your equation to estimate the population of Ontario in the year 2041

3. An elementary school currently has a population of 550 students. It has been estimated that the population of students at the school will decrease by 1.1% each year.

a) Find an equation that will give the number of students at the school each year.
b) Use your equation to predict the student population of the school in: 1 year, 5 years and 10 years.

4. The population of deer in a certain area in the year 2008 was 240. It was estimated that the population of deer was decreasing at a rate of 2% per year.

a) Predict the number of deer in the area in the year 2015.
b) Predict the number of deer in the area in the year 2000.

5. Suppose the price of gasoline increases by approximately 6.5% each year. In the year 2012 (at present) the price of gasoline is $1.20 per litre.

a) Define an equation that gives the price of gas each year.
b) Use your equation to estimate the price of gasoline in the years: 2015, 2020 and 2030.

6. Jake has a bank account that collects interest. The balance of the savings account at the end of each year is shown below. The account balance grows exponentially. Calculate the balance in the account after 10 years.

Year	Balance
0	$5000
1	$5250
2	$5512.50

ANSWERS

1. a) 2.238 million b) 1.4% c) 3 million **2.** a) $y = 10.1(1.0125)^x$ b) 18.8 million **3.** a) y = $550(.989)^x$ b) 544, 520, 492
4. a) 208 b) 282 **5.** a) y = $1.2(1.065)^x$ b) $1.45, $1.99, $3.73 **6.** $8144.47