MCF3M Unit 3, Lesson 5

Completing the Square

Today we will look at completing the square – an algebraic technique for converting a quadratic equation from standard form to ~~factored~~ vertex form.

$y = x^2 + 6x - 1 \quad \rightarrow \quad y = (x+3)^2 - 10$

Complete the square

Warm-up

Factor each of the following:

$ax^2 + bx + c$

$\left(\frac{b}{2}\right)^2$

$x^2 + 6x + 9$

$= (x+3)(x+3)$

$= (x+3)^2$

$x^2 - 4x + 4$

$= (x-2)(x-2)$

$= (x-2)^2$

$x^2 - 10x + 25$

$= (x-5)(x-5)$

$= (x-5)^2$

Now, can you figure out what value would "complete the square" for each of the following? (Then, factor)

$x^2 + 12x + \underline{36} = (x+6)^2$ $\quad \frac{12}{2} = 6$, $6^2 = 36$

$x^2 - 8x + \underline{16} = (x-4)^2$ $\quad \frac{-8}{2} = -4$, $(-4)^2 = 16$

$x^2 - 2x + \underline{1} = (x-1)^2$ $\quad \frac{-2}{2} = -1$, $(-1)^2 = 1$

$x^2 + 3x + \underline{\frac{9}{4}} = (x+\frac{3}{2})^2$ $\quad \frac{3}{2}$, $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

The technique above can be used to change an equation in standard form, to ~~factored~~ vertex form. We will try it below for some simple trinomials (coefficient of ~~one on x^2 term~~)

$f(x) = x^2 + 6x - 1$

$= x^2 + 6x + 9 - 1 - 9$

$= (x+3)^2 - 1 - 9$

$f(x) = (x+3)^2 - 10$

$\frac{6}{2} = 3$, $3^2 = 9$

$y = x^2 - 10x - 3$

$= x^2 - 10x + 25 - 3 - 25$

$y = (x-5)^2 - 28$

$\frac{-10}{2} = -5$, $(-5)^2 = 25$

Now you try the next two...

$f(x) = x^2 + 4x + 1$

$g(x) = x^2 - 12x + 7$

These two might be a little more difficult...

$y = x^2 + x - 3$

$= x^2 + x + \frac{1}{4} - 3 - \frac{1}{4}$

$= (x + \frac{1}{2})^2 - 3\frac{1}{4}$

$\frac{1}{2}$

$(\frac{1}{2})^2 = \frac{1}{4}$

$f(x) = x^2 - 3x$

$= x^2 - 3x + \frac{9}{4} - \frac{9}{4}$

$= (x - \frac{3}{2})^2 - \frac{9}{4}$

$\frac{-3}{2}$

$(-\frac{3}{2})^2 = \frac{9}{4}$

More Examples

Find the domain and range of the quadratic function $f(x) = x^2 - 6x + 5$. Graph the function below.

↳ D: $\{x \in \mathbb{R}\}$

To determine the range, find the vertex.

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$f(x) = x^2 - 6x + 9 + 5 - 9$

$= (x - 3)^2 - 4$

↳ vertex @ $(3, -4)$

↑ range value (minimum because graph opens up)

R: $\{y \in \mathbb{R} \mid y \geq -4\}$