4.2 Combining Functions Algebraically

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FOCUS Write the equations of functions that are the sum, difference, product, or quotient of other functions, then determine their domains and ranges.

Get Started

Given $f(x) = 3 - 4x + 5x^2$, determine each value:

$f(0)$

$= 3 - 4(0) + 5(0)^2$

$= 3$

$f(2)$

$= 3 - 4(2) + 5(2)^2$

$= 3 - 8 + 20$

$= 15$

$f(-1)$

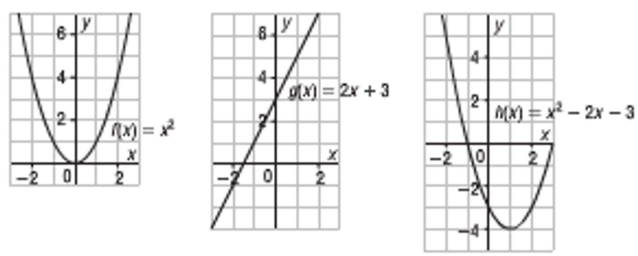
$= 3 - 4(-1) + 5(-1)^2$

$= 3 + 4 + 5$

$= 12$

Construct Understanding

For the graphs below, how is the function $h(x)$ a combination of the functions $f(x)$ and $g(x)$? Use algebra to verify the answer.



$h(x) = x^2 - 2x - 3$

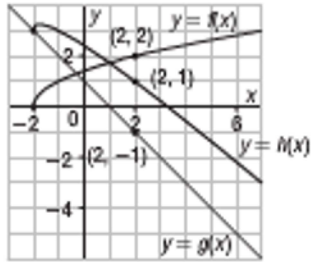
$= x^2 - (2x + 3)$

$= f(x) - g(x)$

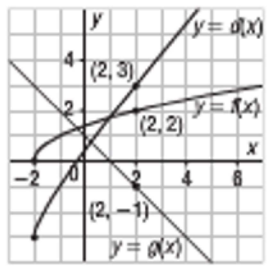
Given two functions $f(x)$ and $g(x)$, another function can be defined by adding, subtracting, or multiplying $f(x)$ and $g(x)$.

Consider coordinates of corresponding points.

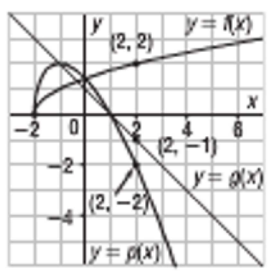
- *Adding functions*
  The sum of functions $f(x)$ and $g(x)$ is $f(x) + g(x)$; this may also be written as $(f + g)(x)$.
  For $h(x) = f(x) + g(x)$, to determine the value of $h(2)$, add $f(2)$ and $g(2)$.
  From the graph, $f(2) + g(2) = 2 - 1$
  $= 1$
  $= h(2)$
  The domain of $f(x)$ is: $x \geq -2$
  The domain of $g(x)$ is: $x \in \mathbb{R}$
  The domain of $h(x)$ is: $x \geq -2$



- *Subtracting functions*
  The difference of functions $f(x)$ and $g(x)$ is $f(x) - g(x)$; this may also be written as $(f - g)(x)$.
  For $d(x) = f(x) - g(x)$, to determine the value of $d(2)$, subtract $g(2)$ from $f(2)$.
  From the graph, $f(2) - g(2) = 2 - (-1)$
  $= 3$
  $= d(2)$
  The domain of $f(x)$ is: $x \geq -2$
  The domain of $g(x)$ is: $x \in \mathbb{R}$
  The domain of $d(x)$ is: $x \geq -2$



- *Multiplying functions*
  The product of functions $f(x)$ and $g(x)$ is $f(x) \cdot g(x)$; this may also be written as $(f \cdot g)(x)$.
  For $p(x) = f(x) \cdot g(x)$, to determine the value of $p(2)$, multiply $f(2)$ and $g(2)$.
  From the graph, $f(2) \cdot g(2) = (2)(-1)$
  $= -2$
  $= p(2)$
  The domain of $f(x)$ is: $x \geq -2$
  The domain of $g(x)$ is: $x \in \mathbb{R}$
  The domain of $p(x)$ is: $x \geq -2$



The graphs above illustrate that when two functions, $f(x)$ and $g(x)$, are added, subtracted, or multiplied, the domain of the new function is the set of values of x that are common to the domains of $f(x)$ and $g(x)$.

Example 1 Writing the Equation of a Function that Is a Combination of Functions

Use $f(x) = \sqrt{x}$ and $g(x) = x - 3$.

a) State the domain and range of $f(x)$ and of $g(x)$.

b) Given $d(x) = f(x) - g(x)$, write an explicit equation for $d(x)$, then determine its domain and range.

c) Given $p(x) = f(x) \cdot g(x)$, write an explicit equation for $p(x)$, then determine its domain and range.

SOLUTION

a) $f(x) = \sqrt{x}$

The domain of $f(x)$ is $x \geq 0$ and the range is $y \geq 0$.

$g(x) = x - 3$

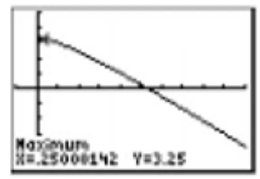
The domain of $g(x)$ is $x \in \mathbb{R}$ and the range is $y \in \mathbb{R}$.

b) In $d(x) = f(x) - g(x)$, substitute: $f(x) = \sqrt{x}$ and $g(x) = x - 3$

So, an equation is:

$d(x) = \sqrt{x} - (x - 3)$, or

$d(x) = \sqrt{x} - x + 3$



The domain of $f(x)$ is $x \geq 0$ and the domain of $g(x)$ is $x \in \mathbb{R}$, so the domain of $d(x)$ is $x \geq 0$.

For the range, use technology to graph $d(x) = \sqrt{x} - x + 3$

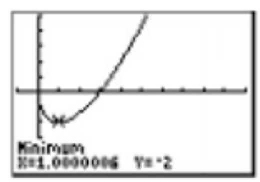
Use 2nd TRACE 4 to determine the maximum value: $y = 3.25$

So, the range of $d(x)$ is: $y \leq 3.25$

c) In $p(x) = f(x) \cdot g(x)$, substitute:

$f(x) = \sqrt{x}$ and $g(x) = x - 3$

So, an equation is: $p(x) = \sqrt{x}(x - 3)$



By the same reasoning as in part b, the domain of $p(x)$ is $x \geq 0$. For the range, use technology to graph $p(x) = \sqrt{x}(x - 3)$

Use 2nd TRACE 3 to determine the minimum value: $y = -2$

So, the range of $p(x)$ is: $y \geq -2$

Check Your Understanding

1. Use $f(x) = x + 2$ and $g(x) = |x|$.

a) State the domain and range of $f(x)$ and of $g(x)$.

b) Given $h(x) = f(x) + g(x)$, write an explicit equation for $h(x)$, then determine its domain and range.

c) Given $p(x) = f(x) \cdot g(x)$, write an explicit equation for $p(x)$, then determine its domain and range.

a) $f(x)$: D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$

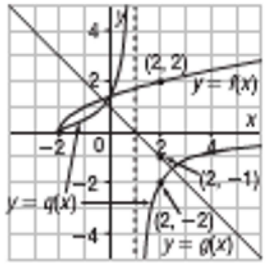
$g(x)$: D: $x \in \mathbb{R}$
R: $y \geq 0$

b) $h(x) = f(x) + g(x)$
$= x + 2 + |x|$
D: $x \in \mathbb{R}$
R: $y \geq 2$

c) $p(x) = f(x) \cdot g(x)$
$= (x+2) \cdot |x|$
D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$

Given two functions $f(x)$ and $g(x)$, another function can be defined by dividing $f(x)$ and $g(x)$.

The quotient of functions $f(x)$ and $g(x)$ is $f(x) \div g(x)$; this may also be written as $(f \div g)(x)$.



Given $q(x) = \frac{f(x)}{g(x)}$, consider coordinates of corresponding points.
To determine the value of $q(2)$, divide $f(2)$ by $g(2)$.

From the graph, $\frac{f(2)}{g(2)} = \frac{2}{-1}$

$= -2$

$= q(2)$

The domain of $f(x)$ is: $x \geq -2$; the domain of $g(x)$ is: $x \in \mathbb{R}$
The domain of $q(x)$ is: $x \geq -2, x \neq 1$

The graph above illustrates that the domain of the function $y = \frac{f(x)}{g(x)}$ is restricted to those values of x for which $g(x) \neq 0$, and for which $f(x)$ and $g(x)$ are defined.

Example 2 Writing the Equation of a Function that Is a Quotient of Functions

Use $f(x) = \sqrt{x}$ and $g(x) = (x - 3)^2$.

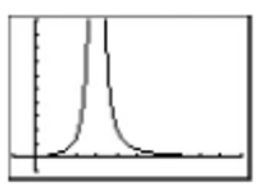
a) State the domain and range of $f(x)$ and of $g(x)$.

b) Given $q(x) = \frac{f(x)}{g(x)}$, write an explicit equation for $q(x)$, then determine its domain and range.

SOLUTION

a) $f(x) = \sqrt{x}$
The domain of $f(x)$ is $x \geq 0$ and the range is $y \geq 0$.
$g(x) = (x - 3)^2$
$g(x)$ is a quadratic function whose graph opens up and has vertex $(3, 0)$; the domain is $x \in \mathbb{R}$ and the range is $y \geq 0$.

b) $q(x) = \frac{f(x)}{g(x)}$



Substitute: $f(x) = \sqrt{x}$ and $g(x) = (x - 3)^2$

$q(x) = \frac{\sqrt{x}}{(x - 3)^2}$

Both $f(x)$ and $g(x)$ are defined for $x \geq 0$, and $g(x) = 0$ when $x = 3$.
So, the domain of $q(x)$ is: $x \geq 0, x \neq 3$

For the range, use technology to graph: $q(x) = \frac{\sqrt{x}}{(x - 3)^2}$

Use TRACE or 2nd GRAPH for TABLE to determine that the range of $q(x)$ is: $y \geq 0$

Check Your Understanding

2. Use $f(x) = \sqrt{x}$ and $g(x) = x - 2$.

a) State the domain and range of $f(x)$ and of $g(x)$.

b) Given $q(x) = \frac{f(x)}{g(x)}$, write an explicit equation for $q(x)$, then determine its domain and range.

a) $f(x)$: D: $x \geq 0$
R: $y \geq 0$

$g(x)$: D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$

b) $q(x) = \frac{\sqrt{x}}{x-2}$

D: $x \geq 0, x \neq 2$
R: $y \in \mathbb{R}$

Assignment:
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