# Rational Exponents

Today we will answer questions such as:

What does $2^{\frac{1}{2}}$ (2 to the power of ½")mean? What about $4^{\frac{1}{2}}$? What about $8^{\frac{1}{3}}$ or $27^{\frac{2}{3}}$?

We can try answer these questions a few different ways.

1) Using patterns.

$\sqrt{4} = 2$

$4^0 = 1$ (×2)
$4^{1/2} = 2$ (×2) (×4)
$4^1 = 4$ (×2)
$4^{3/2} = 8$ (×2) (×4)
$4^2 = 16$ (×2)

$\sqrt{9} = 3$

$9^0 = 1$ (×3)
$9^{1/2} = 3$ (×3) (×9)
$9^1 = 9$ (×3)
$9^{3/2} = 27$ (×3) (×9)
$9^2 = 81$ (×3)

$2^0 = 1$ (×√2)
$2^{1/2} = \sqrt{2}$ (×√2) (×2)
$2^1 = 2$ (×√2)
$2^{3/2} = 2\sqrt{2}$ (×√2) (×2)
$2^2 = 4$ (×√2)

2) Using exponent laws.

$\frac{1}{2} + \frac{1}{2} = 1$

$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$

$2 \times 2 = 4$

$\Rightarrow 4^{1/2} = 2$

$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^1$

$\sqrt{5} \times \sqrt{5} = 5$

$\Rightarrow 5^{1/2} = \sqrt{5}$

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^1$

$2 \times 2 \times 2 = 8$

$\Rightarrow 8^{1/3} = 2$

$8^{1/3} = \sqrt[3]{8} = 2$

$32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} = 32^1$

$2 \times 2 \times 2 \times 2 \times 2 = 32$

$\Rightarrow 32^{1/5} = \sqrt[5]{32} = 2$

**Examples:** Evaluate the following:

$81^{\frac{1}{2}} = \sqrt{81} = 9$

$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$16^{-\frac{1}{2}} = \frac{1}{16^{1/2}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

How might we evaluate the following?

$81^{\frac{3}{4}} = (81^{1/4})^3$ or $(81^3)^{1/4}$

$= (\sqrt[4]{81})^3$ same $\sqrt[4]{81^3}$

$= (3)^3$

$= 27$

**More Examples:**

$8^{\frac{2}{3}}$

$= (8^{1/3})^2$

$= (\sqrt[3]{8})^2$

$= (2)^2$

$= 4$

$\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$= \left[\left(\frac{9}{16}\right)^{1/2}\right]^3$

$= \left(\sqrt{\frac{9}{16}}\right)^3$

$= \left(\frac{3}{4}\right)^3$

$= \frac{27}{64}$

$\left(\frac{9}{16}\right)^{\frac{-3}{2}}$

$= \left(\frac{16}{9}\right)^{3/2}$

$= \left(\left(\frac{16}{9}\right)^{1/2}\right)^3$

$= \left(\sqrt{\frac{16}{9}}\right)^3$

$= \left(\frac{4}{3}\right)^3$

$= \frac{64}{27}$

$(-27)^{\frac{2}{3}}$

$= \left((-27)^{1/3}\right)^2$

$= \left(\sqrt[3]{-27}\right)^2$

$= (-3)^2$

$= 9$

$(-8)^{2/3}$

$= \left((-8)^{1/3}\right)^2$

$= \left(\sqrt[3]{-8}\right)^2$

$= (-2)^2$

$= 4$