

Problem Solving with the Quadratic Formula

Example 1

A model rocket is launched upwards with a velocity of 16 m/s. The equation that models the height of the rocket is $h(t) = -4.9t^2 + 16t + 1.1$ where $h(t)$ is the height (in metres) after t seconds.

- a) Find the initial height of the rocket.

$$1.1 \text{ m}$$

- b) How long is the rocket in the air for? (round to 2 decimal places)

$$\begin{aligned} x &= \frac{-16 \pm \sqrt{16^2 - 4(-4.9)(1.1)}}{2(-4.9)} \\ &= \frac{-16 \pm \sqrt{256 + 21.56}}{-9.8} \\ &= \frac{-16 \pm \sqrt{277.56}}{-9.8} \\ &\approx -0.07, 3.33 \end{aligned}$$

The rocket is in the air for 3.33 seconds.

- c) Does the rocket ever reach a height of 60m? If so, for how long?

$$-4.9t^2 + 16t + 1.1 = 60$$

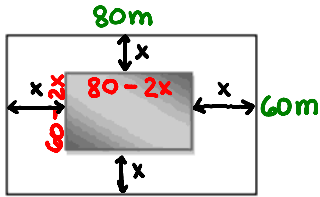
$$-60 \quad -60$$

$$\begin{aligned} -4.9t^2 + 16t - 58.9 &= 0 \quad \rightarrow \text{check discriminant: } 16^2 - 4(-4.9)(-58.9) \\ &= 256 - 1154.44 \\ &= -898.44 \end{aligned}$$

The rocket does not reach a height of 60 m.

Example 2

A factory is to be built on a lot that measures 80 m by 60 m. A lawn of uniform width, equal to that of the factory to the area of the factory, must surround it. How wide is the strip of the lawn?



$$\text{Area of factory} = \frac{1}{2}(80)(60) = 2400 \text{ m}^2$$

$$(60-2x)(80-2x) = 2400$$

$$4800 - 120x - 160x + 4x^2 = 2400$$

$$4x^2 - 280x + 4800 = 2400$$

$$-2400 \quad -2400$$

$$4x^2 - 280x + 2400 = 0 \rightarrow x = \frac{280 \pm \sqrt{(-280)^2 - 4(4)(2400)}}{2(4)}$$

$$= \frac{280 \pm \sqrt{40000}}{8}$$

$$= \frac{280 \pm 200}{8}$$

$$= 60, 10$$

The width of the lawn is 10m.

↑ doesn't make sense

Example 3

A triangle has an area of 1250 cm^2 . Its height is 10 cm longer than its base. Find its dimensions.

$$2 \cdot \frac{\text{base} \cdot \text{height}}{2} = 1250 \cdot 2$$

$$x(x+10) = 2500$$

$$x^2 + 10x = 2500$$

$$x^2 + 10x - 2500 = 0 \rightarrow x = \frac{-10 \pm \sqrt{10^2 - 4(1)(-2500)}}{2(1)}$$

$$= \frac{-10 \pm \sqrt{10100}}{2}$$

$$= 45.25, -55.25$$

↑ doesn't make sense

The base is 45.25 cm and the height is 55.25 cm.