

An annuity is a set of regular payments, such as a car loan, mortgage, or the following scenario.

Alan's parents decide they would like to contribute towards saving for his post-secondary education. They will **contribute \$1200 at the end of each school year (June)** towards an annuity that pays interest at **3%/a compounded annually**. They begin the payments at the end of grade 9 and finish the end of grade 12. (4 years)

How much money will be saved in Alan's education account in June of his grade 12 year?

Timeline Solution:

Year:	June Gr 9	June Gr 10	June Gr 11	June Gr 12
Deposit:	\$1200	\$1200	\$1200	\$1200
			$\times 1.03$	\$1236
		$\times (1.03)^2$		\$1273.08
	$\times (1.03)^3$			\$1311.27
				Total: \$5020.35

Future Value of an Annuity Formula:

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

future value \rightarrow FV
 R \leftarrow payments
 n \leftarrow number of compounding periods
 i \leftarrow interest per compounding period

Solve the question above using the formula.

$$\frac{1200[(1.03)^4 - 1]}{0.03} = 5020.35$$

Example: In order to save for his own post-secondary education, Aaron puts away \$100 at the end of each month in an annuity paying 4%/a compounded monthly. If he does this for four years, what is the final value of the annuity?

$$\frac{100\left[\left(1 + \frac{0.04}{12}\right)^{48} - 1\right]}{\frac{0.04}{12}} = 5195.96$$

The final value is \$5195.96

How much interest did Aaron earn?

$$5195.96 - 100(48) = 395.96$$

Aaron earned \$395.96.

↙ every two weeks

Example: Gage is 35 years old and starts saving for retirement. He is paid bi-weekly and \$100 from each paycheque is deposited into an RRSP that pays 3.5%/a compounded bi-weekly. How much money will he have if he wants to retire at age 60?

$$25 \times 26 = 650 \text{ deposits}$$

$$\frac{100 \left[\left(1 + \frac{0.035}{26} \right)^{650} - 1 \right]}{\frac{0.035}{26}} = 103811.62$$

Gage has \$103811.62.

Finding Payments

Ethan wants to purchase a condo in Whistler. The condo costs \$350,000. He wants to save up enough money to purchase the condo in 10 years. He can invest the money into a savings account that pays 7.5%/a compounded monthly. Condo prices in Whistler are increasing by 2.5% per year. How much will Ethan have to save every month to pay for the condo in cash?

$$\text{future value} = 350000(1.025)^{10} = \$448030$$

$$\frac{R \left[\left(1 + \frac{0.075}{12} \right)^{120} - 1 \right]}{\frac{0.075}{12}} = 448030$$
$$\frac{177.93 R}{177.93} = \frac{448030}{177.93}$$
$$R = 2518$$

Ethan has to save \$2518 every month.

If Ethan does purchase the condo, how much interest did he manage to make while saving money?

$$448030 - 2518(120) = 145870$$

Ethan makes \$145870 in interest.

Practice:

1. An ambitious grade 9 student decides to start saving for their University education. At the end of every month the student deposits \$150 into a savings account that pays 3.5%/a compounded monthly. After exactly 4 years, how much money would the student have saved?

$$\frac{150 \left[\left(1 + \frac{0.035}{12} \right)^{48} - 1 \right]}{\frac{0.035}{12}} = 7716.31$$

The student has \$7716.31 saved.

2. Ms Kob wants to save for her retirement. At the end of every month she puts \$100 into an RRSP that pays 6.5%/a compounded monthly.

a) How much will she have if she retires in 30 years?

b) Suppose that instead of deciding to put away \$100 every month, Ms Kob decides she wants to calculate what monthly payments will allow her to retire with \$500,000. What monthly payments will she need to make?

3. You are planning to start university in 14 months. In exactly 15 months you will need \$8000 to pay for tuition, residence and books. How much money would you need to save (at the end of every month starting with the end of this month) to arrive at your goal? Suppose that you can invest the money at 2%/a compounded monthly.