

3.3 Factoring Trinomials Part I

Math 10

Factoring Trinomials

The following method is called **factoring by grouping**:

1. Remove the GCF from the first two terms, and the GCF from the last two terms.
2. Identify the common factor (which is a binomial) and rewrite the expression in factored form.

Example: $x^2 + 3x + 4x + 12$

$$1. \quad x(x+3) + 4(x+3)$$

$$2. \quad (x+3)(x+4)$$

Apply this method to the following expressions.

$$\begin{aligned} m^2 - 5m + 3m - 15 \\ &= m(m-5) + 3(m-5) \\ &= (m-5)(m+3) \end{aligned}$$

$$\begin{aligned} a^2 - ac + 2ac - 2c^2 \\ &= a(a-c) + 2c(a-c) \\ &= (a-c)(a+2c) \end{aligned}$$

$$\begin{aligned} y^2 + 6y - 2y - 12 \\ &= y(y+6) - 2(y+6) \\ &= (y+6)(y-2) \end{aligned}$$

$$\begin{aligned} x^2 - 4x - x + 4 \\ &= x(x-4) - 1(x-4) \\ &= (x-4)(x-1) \end{aligned}$$

Now we will apply this to trinomials. These trinomials have already been factored. Do you see any relationship between the trinomial and its factored form?

$$x^2 + 7x + 6 = (x + 1)(x + 6)$$

$$x^2 + 6x + 8 = (x + 4)(x + 2)$$

$$x^2 - 11x + 24 = (x - 3)(x - 8)$$

$$x^2 - 9x + 14 = (x - 2)(x - 7)$$

$$x^2 + 3x - 10 = (x - 2)(x + 5)$$

$$x^2 - 9x + 8 = (x - 1)(x - 8)$$

$$x^2 - 5x - 6 = (x + 1)(x - 6)$$

$$x^2 + 4x - 12 = (x - 2)(x + 6)$$

To factor the following trinomials:

1. Determine the two numbers which multiply to the constant and sum to the "middle" term.
2. Split the middle term. (There should now be four terms.)
3. Remove the GCF from the first two terms, and the GCF from the last two terms.
4. Identify the common factor (which is a binomial) and rewrite the expression in factored form.

$$\begin{aligned}
 x^2 + 7x + 6 &= (x+6)(x+1) \\
 \frac{\cdot 6}{2 \cdot 3} & \frac{\cdot 7}{2+3} \times \\
 6 \cdot 1 & 6+1 \checkmark \\
 &= x^2 + 6x + 1x + 6 \\
 &= x(x+6) + 1(x+6) \\
 &= (x+6)(x+1) \\
 x^2 - 11x + 24 &= (x-8)(x-3) \\
 \frac{\cdot 24}{12 \cdot 2} & \frac{-11}{12+2} \times \\
 8 \cdot 3 & 8+3 \\
 (-3) \cdot (-3) & -8+(-3) = -11 \checkmark \\
 &= x^2 - 8x - 3x + 24 \\
 &= x(x-8) - 3(x-8)
 \end{aligned}$$

$$\begin{aligned}
 x^2 + 3x - 10 &= (x-2)(x+5) \\
 \frac{\cdot -10}{-2 \cdot 5} & \frac{\cdot 3}{-2+5} \checkmark \\
 &= x^2 - 2x + 5x - 10 \\
 &= x(x-2) + 5(x-2) \\
 &= (x-2)(x+5)
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 5x - 6 &= (x-6)(x+1) \\
 \frac{\cdot -6}{-6 \cdot 1} & \frac{-5}{\checkmark} \\
 &= x^2 - 6x + 1x - 6 \\
 &= x(x-6) + 1(x-6) \\
 &= (x-6)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 x^2 + 6x + 8 &= (x+2)(x+4) \\
 \frac{\cdot 8}{2 \cdot 4} & \frac{\cdot 6}{2+4} \checkmark \\
 &= x^2 + 2x + 4x + 8 \\
 &= x(x+2) + 4(x+2) \\
 &= (x+2)(x+4) \\
 x^2 - 9x + 14 &= (x-7)(x-2) \\
 \frac{\cdot 14}{7 \cdot 2} & \frac{-9}{7+2} \\
 -7 \cdot (-2) & -7+(-2) \checkmark \\
 &= x^2 - 7x - 2x + 14 \\
 &= x(x-7) - 2(x-7)
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 9x + 8 &= (x-8)(x-1) \\
 \frac{\cdot 8}{-8 \cdot (-1)} & \frac{-9}{\checkmark} \\
 &= x^2 - 8x - 1x + 8 \\
 &= x(x-8) - 1(x-8) \\
 &= (x-8)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 x^2 + 4x - 12 &= (x+6)(x-2) \\
 \frac{\cdot -12}{-4 \cdot 3} & \frac{\cdot 4}{-4+3} \times \\
 6 \cdot (-2) & 6+(-2) = 4 \checkmark \\
 &= x^2 + 6x - 2x - 12 \\
 &= x(x+6) - 2(x+6) \\
 &= (x+6)(x-2)
 \end{aligned}$$