3.2 Reflecting Graphs of Functions
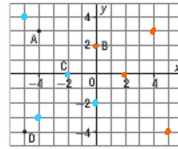
3.2 Reflecting Graphs of Functions

FOCUS Relate changes in the equation of a function to reflections of its graph in the x- and y-axes.

Get Started

Write the coordinates of each point.

$A(-4,3)$ $C(-2,0)$
$B(0,2)$ $D(-5,-4)$

Reflect each point in the x-axis, then write the coordinates of the reflection image. What do you notice?

$A'(-4,-3)$ $C'(-2,0)$
$B'(0,-2)$ $D'(-5,4)$

y-values switch signs
(0 stays the same)
x-values don't change

Reflect each given point in the y-axis, then write the coordinates of the reflection image. What do you notice?

$A''(4,3)$ $C''(2,0)$
$B''(0,2)$ $D''(5,-4)$

x-values switch signs
(0 stays the same)
y-values don't change

Construct Understanding

On each grid below, reflect the original graph in the x-axis and in the y-axis. For each reflection, write an equation for the image graph. Describe any patterns in points on the graphs and the equations of the graphs.
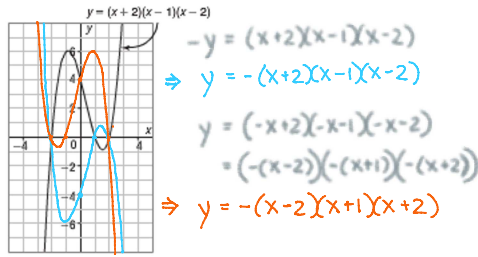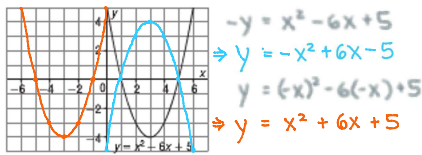Use the patterns to predict equations of the function $f(x) = |x^2 - 2x|$ after a reflection in the x-axis and in the y-axis.
Use graphing technology to check your predictions.
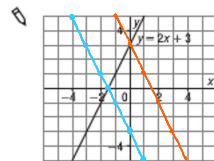
$y = -2x - 3$ replace y with -y
$-y = 2x + 3$
$\Rightarrow y = -2x - 3$

$y = -2x + 3$ replace x with -x
$y = 2(-x) + 3$
$\Rightarrow y = -2x + 3$

DO NOT COPY. ©P

$-y = x^2 - 6x + 5$

$\Rightarrow y = -x^2 + 6x - 5$

$y = (-x)^2 - 6(-x) + 5$

$\Rightarrow y = x^2 + 6x + 5$

$y = (x + 2)(x - 1)(x - 2)$

$-y = (x+2)(x-1)(x-2)$

$\Rightarrow y = -(x+2)(x-1)(x-2)$

$y = (-x+2)(-x-1)(-x-2)$

$= (-(x-2))(-(x+1))(-(x+2))$

$\Rightarrow y = -(x-2)(x+1)(x+2)$

The graph of a function $y = f(x)$ may be reflected in the x-axis or in the y-axis. For example, the graph of $y = -\sqrt{x - 2}$ is the image of the graph of $y = \sqrt{x - 2}$ after a reflection in the x-axis. The graph of $y = \sqrt{-x - 2}$ is the image of the graph of $y = \sqrt{x - 2}$ after a reflection in the y-axis.
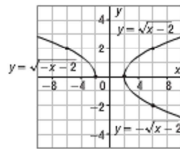
The point $(6, 2)$ lies on the graph of $y = \sqrt{x - 2}$. The corresponding point on the graph of $y = \sqrt{-x - 2}$ is $(-6, 2)$, and the corresponding point on the graph of $y = -\sqrt{x - 2}$ is $(6, -2)$. This example combined with the preceding examples can be generalized as follows.

Reflecting in the Axes

For a function $y = f(x)$

- The graph of $y = -f(x)$ is the image of the graph of $y = f(x)$ after a reflection in the x-axis. A point (x, y) on $y = f(x)$ corresponds to the point $(x, -y)$ on $y = -f(x)$.

- The graph of $y = f(-x)$ is the image of the graph of $y = f(x)$ after a reflection in the y-axis. A point (x, y) on $y = f(x)$ corresponds to the point $(-x, y)$ on $y = f(-x)$.
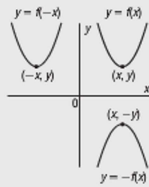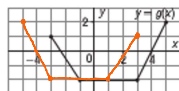
The image of the graph of a function $y = f(x)$ after a reflection in the x-axis or the y-axis is congruent to the graph of $y = f(x)$, but the graphs may have different orientations.

Example 1   Sketching the Graph of a Function after a Reflection

Check Your Understanding

1. Here is the graph of $y = g(x)$. Sketch the image graph after a reflection in the y-axis. State the domain and range of each function.
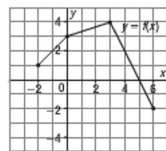
Here is the graph of $y = f(x)$. Sketch the image graph after a reflection in the x-axis. State the domain and range of each function.

$y = g(x)$
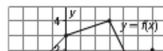
$D: \{-3 \le x \le 5, x \in R\}$

$R: \{-2 \le y \le 2, y \in R\}$

SOLUTION

After a reflection in the x-axis, the image of each point (x, y) on $y = f(x)$ is the point $(x, -y)$ on the image graph.

$y = g(x)$
$D: \{-3 \le x \le 5, x \in \mathbb{R}\}$
$R: \{-2 \le y \le 2, y \in \mathbb{R}\}$

$y = g(-x)$
$D: \{-5 \le x \le 3, x \in \mathbb{R}\}$
$R: \{-2 \le y \le 2, y \in \mathbb{R}\}$

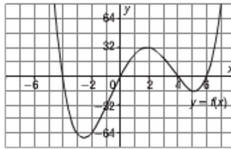**SOLUTION**

After a reflection in the x-axis, the image of each point (x, y) on $y = f(x)$ is the point $(x, -y)$ on the image graph.
Reflect the endpoint of each line segment on $y = f(x)$ in the x-axis. Join corresponding points in order, to form the reflection image.
From the graph, both functions have domain: $-2 \le x \le 6$
The range of $y = f(x)$ is: $-2 \le y \le 4$
The range of the reflection image is: $-4 \le y \le 2$
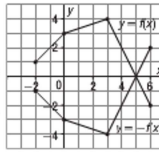
| **Example 2**	**Sketching the Graph of a Polynomial Function after a Reflection**

Here is the graph of $y = f(x)$. Sketch the graph of $y = f(-x)$.
State the domain and range of each function.

Check Your Understanding

2. Here is the graph of $y = g(x)$. Sketch the graph of $y = -g(x)$. State the domain and range of each function.

$y = g(x)$ $D: \{x \in \mathbb{R}\}$
            $R: \{y \in \mathbb{R}\}$

$y = -g(x)$ $D: \{x \in \mathbb{R}\}$
            $R: \{y \in \mathbb{R}\}$

Assignment: p.183
#1, 3-6, 9-11

SOLUTION

The graph of $y = f(-x)$ is the image of the graph of $y = f(x)$ after a reflection in the y-axis. To sketch, use the transformation: (x, y) on $y = f(x)$ corresponds to $(-x, y)$ on $y = f(-x)$.
Estimate the coordinates of points on $y = f(x)$.

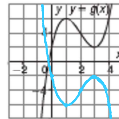| Point on $y = f(x)$	Point on $y = f(-x)$
(−4, 0)	(4, 0)
(−3, −64)	(3, −64)
(−2, −64)	(2, −64)
(0, 0)	(0, 0)
(2, 32)	(−2, 32)
(4, 0)	(−4, 0)
(5, −16)	(−5, −16)
(6, 0)	(−6, 0)
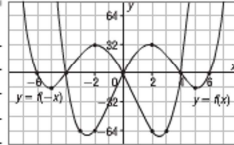
Plot the points, then draw a smooth curve through the points to form the graph of $y = f(-x)$.
Both functions have domain: $x \in \mathbb{R}$
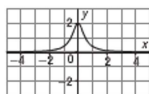Both functions have the same range. The approximate range is:
$y \ge -68$

THINK FURTHER

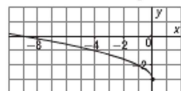When the graph of a function is reflected in an axis, which points are invariant?

✎

3. The graph of

$$y = \frac{1}{-2x^2 - 0.5}$$

was reflected in the *x*-axis and its image is shown. What is an equation of the image?

✎

| | | | | | |

Example 3 | **Writing the Equation of the Graph of a Reflection Image**

The graph of $y = \sqrt{x} - 3$ was reflected in the *y*-axis and its image is shown. What is an equation of the image?

SOLUTION

When the graph of $y = f(x)$ is reflected in the *y*-axis, the equation of its image is $y = f(-x)$.

So, an equation of the image is:

$$y = f(-x)$$
$$y = \sqrt{(-x)} - 3$$
$$y = \sqrt{-x} - 3$$

THINK FURTHER

Suppose the graph of $y = f(x)$ is reflected in the *x*-axis, then its image is reflected in the *y*-axis. What is the equation of the final image?

✎

When the equation of a function is given, the different reflections can be performed on a TI-83 Plus graphing calculator.

For example, to reflect the function in *Example 3*:

Define Y_1 as $\sqrt{x} - 3$.

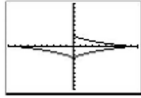To define Y_2 as $-f(x)$, set the cursor to the right of $Y_2 =$.

Press: (-) VARS ▶ 1 1

To define Y_3 as $f(-x)$, set the cursor to the right of $Y_3 =$.

Press: VARS ▶ 1 1 ((-) X,T,Θ,n)

Press: GRAPH
Here is what you should see:

```
Plot1  Plot2  Plot3
\Y1■f(X)-3
\Y2■-Y1
\Y3■Y1(-X)
\Y4=
\Y5=
\Y6=
\Y7=
```

Discuss the Ideas

1. What is the relationship between the coordinates of a point and the coordinates of its image after a reflection in the x-axis and after a reflection in the y-axis?

2. What strategy do you use to remember which equation, $y = -f(x)$ or $y = f(-x)$, corresponds to a reflection in each axis?

Exercises

A

3. Here is the graph of $y = f(x)$. On the same grid, sketch its image after each reflection.

a) a reflection in the x-axis