The Discriminant of the Quadratic Formula

Warm-up

Use the quadratic formula to solve each equation below.

a) $2x^2 - 5x - 9 = 0$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-9)}}{2(2)}$

$= \frac{5 \pm \sqrt{25 + 72}}{4}$

$= \frac{5 \pm \sqrt{97}}{4}$

$\doteq 3.71, -1.21$

b) $x^2 + 12x + 36 = 0$

$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(36)}}{2(1)}$

$= \frac{-12 \pm \sqrt{144 - 144}}{2}$

$= \frac{-12 \pm 0}{2}$

$= -6$

c) $2x^2 + x + 5 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(5)}}{2(2)}$

$= \frac{-1 \pm \sqrt{1 - 40}}{4}$

$= \frac{-1 \pm \sqrt{-39}}{4}$

no solution

The examples above illustrate all of the possibilities when solving a quadratic equation. We could have 1 solution, 2 solutions or no solutions.

In fact a certain part of the quadratic formula (called the **discriminant**) tells us how many solutions we can expect.

In general:

If $b^2 - 4ac > 0$ then there are 2 solutions.

If $b^2 - 4ac = 0$ then there is 1 solution.

If $b^2 - 4ac < 0$ then there is no solution.

How many solutions will each of the following have? (You do not need to solve)

$4x^2 + x - 1 = 0$ $b^2 - 4ac$

$1^2 - 4(4)(-1) = 1 + 16 = 17 > 0$

$\therefore$ 2 solutions

$x^2 + 40 = 2x$

$x^2 - 2x + 40 = 0$

$(-2)^2 - 4(1)(40)$

$= 4 - 160 < 0$

$\therefore$ no solution

When solving a quadratic equation it is never a bad idea to check the discriminant first. After all, it is part of the formula anyways!

Determine the number of solutions to each quadratic equation, then solve (if you can!)

$5x^2 = x + 1$ $5x^2 - x - 1 = 0$

$(-1)^2 - 4(5)(-1) = 1 + 20 = 21$

$\rightarrow$ 2 solutions

$x = \frac{1 \pm \sqrt{21}}{10}$

$\doteq 0.56, -0.36$

$5x^2 - 2x + 6 = 0$

$(-2)^2 - 4(5)(6)$

$= 4 - 120$

$= -116$

no solution

How can the discriminant tell us whether a quadratic equation can be solved by ***factoring?*** Give an example.

If the value of the discriminant is a perfect square, then the equation can be solved by factoring.

The Discriminant and the x-intercepts/Zeroes

Recall that to find the x-intercepts (zeroes) of a quadratic function we must first set the equation to zero and solve. Hence, the discriminant can tell us how many zeroes a parabola will have.

Find the number of zeroes (x-intercepts) for each parabola below.

a) $y = x^2 - 2x + 9$

$x^2 - 2x + 9 = 0$

$(-2)^2 - 4(1)(9)$

$= 4 - 36$

$= -32 < 0$ $\rightarrow$ no x-intercepts

b) $f(x) = 2x^2 - 9x$

$2x^2 - 9x = 0$

$(-9)^2 - 4(2)(0)$

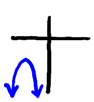
$= 81 > 0$

$\rightarrow$ 2 x-intercepts

c) $g(x) = -2(x+3)^2 - 8$

opens down

vertex is below the x-axis



$\rightarrow$ no x-intercepts

d) $g(x) = (x-2)^2$

opens up

vertex is on x-axis

$\rightarrow$ 1 x-intercept

Sometimes a solution to a quadratic equation is called a **real root.**