The Quadratic Formula

Warm-up

Find the x-intercepts (zeroes) for the parabola given by the equation $f(x) = x^2 - 8x + 15$.

$f(x) = (x-5)(x-3)$

x-int = 5, 3

Now, try to find the x-intercepts of the parabola given by the equation $f(x) = x^2 - 5x - 1$.

not factorable!

There are actually 2 x-intercepts for the above function. How can we verify this?

y-int is below the x-axis + the graph opens up

So how do we find them?

There is a formula that will give the roots (solutions) to any quadratic equation of the form $ax^2 + bx + c = 0$.

The solution(s) to the quadratic equation $ax^2 + bx + c = 0$ are always given by the formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (the quadratic formula).

Example. Use the quadratic formula to solve the equation $0 = x^2 - 5x - 1$.

$a = 1 \quad b = -5 \quad c = -1$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)}$

$= \frac{5 \pm \sqrt{25 + 4}}{2}$

$= \frac{5 \pm \sqrt{29}}{2}$

$x = \frac{5 + \sqrt{29}}{2} \doteq 5.19$

$x = \frac{5 - \sqrt{29}}{2} \doteq -0.19$

The solutions to a quadratic equation are often irrational numbers. If they are not, then likely the equation could have been solved by factoring.

Examples: Solve each of the following.

a) $2x^2 - 9x + 1 = 0$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81 - 8}}{4}$$

$$= \frac{9 \pm \sqrt{73}}{4}$$

$$\doteq 4.39, 0.11$$

b) $x^2 - 6x - 27 = 0$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 + 108}}{2}$$

$$= \frac{6 \pm \sqrt{144}}{2}$$

$$= \frac{6 \pm 12}{2}$$

$$= 9, -3$$

Be careful if the equation is not set to zero.

Solve $-3x^2 - 5 = 8x$

$$-3x^2 - 8x - 5 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(-3)(-5)}}{2(-3)}$$

$$= \frac{8 \pm \sqrt{64 - 60}}{-6}$$

$$= \frac{8 \pm \sqrt{4}}{-6} = \frac{8 \pm 2}{-6} = -\frac{10}{6}, -1$$

Some tricks can be used if fractions or decimals are involved.

Solve $-3.2x^2 - 1.9x + 2.3 = 0$ multiply by 10

$$-32x^2 - 19x + 23 = 0$$

$$x = \frac{19 \pm \sqrt{(-19)^2 - 4(-32)(23)}}{2(-32)}$$

$$= \frac{19 \pm \sqrt{361 + 2944}}{-64}$$

$$= \frac{19 \pm \sqrt{3305}}{-64}$$

$$\doteq -1.20, 0.60$$

$12 \cdot \frac{1}{2}x^2 - \frac{12}{1} \cdot \frac{2}{3}x = \frac{12}{1} \cdot \frac{-3}{4}$ multiply by LCD

$$\frac{12}{2}x^2 - \frac{24}{3}x = \frac{-36}{4}$$

$$6x^2 - 8x = -9$$

$$6x^2 - 8x + 9 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(9)}}{2(6)}$$

$$= \frac{8 \pm \sqrt{64 - 216}}{12}$$

$$= \frac{8 \pm \sqrt{-152}}{12}$$ no solution