

Interest is a fee that is charged by a lender to a borrower for the right to use the borrowed funds. There are two types of interest:

1. Simple Interest: The same amount of interest is earned each term.

Simple interest is calculated using the formula:

$$I = Prt$$

Handwritten annotations for the formula $I = Prt$:
 - I : interest earned
 - P : principal (initial amount)
 - r : interest rate
 - t : time

Example: Ben puts \$3000 into a GIC savings account that collects 4.5%/a simple interest.

- a) How much interest does he earn in 4 years?

$$I = 3000(0.045)(4) = \$540$$

↑ annual rate
"per annum"

- b) What is the total value of the GIC in 4 years?

$$3000 + 540 = \$3540$$

2. Compound Interest: Interest is earned on the accumulated interest along with the principal amount.

Compound interest is calculated using the formula:

$$A = P(1 + i)^n$$

Handwritten annotations for the formula $A = P(1 + i)^n$:
 - A : accumulated amount
 - P : principal
 - i : interest rate
 - n : number of compounding periods

Common Compounding Periods:

- annually (once per year)
- semi-annually (twice per year)
- quarterly (four times per year)
- monthly (twelve times per year)
- daily (365 times per year; 366 times in a leap year)

Example: Find the final value of a \$6000 investment compounded quarterly for 5 years at a rate of 2.5% per annum.

$$A = 6000 \left(1 + \frac{0.025}{4}\right)^{20} = \$6796.25$$

Handwritten annotation: 4 times per year for 5 years

Example: Find the final value of a \$10,000 investment compounded daily for 2 years at a rate of 2.5% per annum. How much interest was earned?

$$A = 10000 \left(1 + \frac{0.025}{365}\right)^{730} = \$10512.69$$

Handwritten annotation: 365 x 2

Compound interest is an example of exponential growth. Simple interest is an example of a linear relation.

1. Crystal inherits \$7600 on her 13th birthday. The money ^{is} invested into a GIC that pays 3.75%/a compounded monthly. How much will it be worth on her 18th birthday?

$$A = 7600 \left(1 + \frac{0.0375}{12} \right)^{60} \leftarrow 5 \times 12 = \$9164.67$$

The GIC will be worth \$9164.67.

2. Liam doesn't pay off \$1000 on his credit card bill. Interest is charged at a rate of 19%/a compounded daily. If he waits one year before paying off the bill, how much will he owe?

$$A = 1000 \left(1 + \frac{0.19}{365} \right)^{365} = 1209.19$$

Liam will owe \$1209.19.

3. Susan will start university in 2 years. How much money must she invest right now (at 2.5%/a compounded monthly) to have exactly \$4000 to pay her tuition?

$$4000 = P \left(1 + \frac{0.025}{12} \right)^{24} \rightarrow P = \frac{4000}{\left(1 + \frac{0.025}{12} \right)^{24}} = 3805.12$$

Susan must invest \$3805.12.

4. Trevor has a savings account that pays interest compounded monthly. He deposits \$5000 into the account. Two years later the account has a balance of \$5152.18. What interest rate did the savings account pay?

$$5152.18 = 5000 \left(1 + \frac{i}{12} \right)^{24}$$

The savings account pays 1.5% interest.

$$\frac{5152.18}{5000} = \left(1 + \frac{i}{12} \right)^{24}$$

$$\left(\frac{5152.18}{5000} \right)^{1/24} = 1 + \frac{i}{12} \rightarrow \left(\frac{5152.18}{5000} \right)^{1/24} - 1 = \frac{i}{12}$$

$$12 \cdot 0.001250031 = \frac{i}{12} \cdot 12$$

$$0.015 = i$$

Practice:

1. You receive an inheritance from a great-aunt and suddenly have \$36000. You decide to invest the money in one of the following three options. For each investment, calculate the interest earned and the total amount of money that you have at the end of the investment period.

- a) 90 day term deposit at 7.5%/a simple interest
- b) 3 year GIC at 12.5%/a compounded semi-annually
- c) 10 year TFSA at 2%/a compounded monthly

2. Your friend just cashed in a 3 year GIC for \$4400. The GIC paid 7.63%/a compounded monthly. How much did your friend originally invest? How much interest did she earn?

3. On the day Elsa was born her parents invested \$2000 in a GIC. The investment has earned 7.5% compounded quarterly. This year, for her birthday, her parents cashed in the GIC and gave her the money. She got \$6566.86. How old is Elsa?