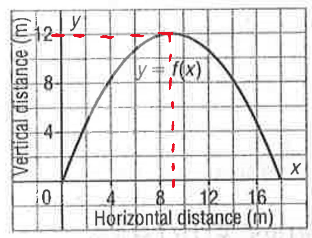
1.5 Modelling and Solving Problems with Polynomial Functions

FOCUS Analyze the graph of a polynomial function to solve a problem.

Get Started

This graph represents the path of a ball kicked into the air.



What was the maximum height of the ball?

12m

How far had the ball travelled horizontally when it reached this height?

9m

What total horizontal distance did the ball travel?

18m

Construct Understanding

> Use graphing technology.
> A certain airline's regulations state that the sum of the length, width, and depth of a piece of carry-on luggage must not exceed 100 cm. Several models of carry-on luggage have length 10 cm greater than their depth.
> Write a polynomial function to represent the volume of this luggage. Graph the function.
> To the nearest cubic centimetre, what is the maximum possible volume of this luggage? What are its dimensions to the nearest tenth of a centimetre? What assumptions did you make?

$L + W + D \leq 100$

↑ $D+10$

$D+10+W+D \leq 100$

$2D + W \leq 90$

$W \leq 90 - 2D$

$V = L \cdot W \cdot D$

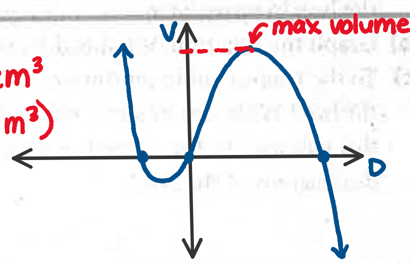
$\leq (D+10)(90-2D)(D)$

leading coefficient $= D \cdot (-2D) \cdot D = -2D^3$

negative cubic function

zeros @ $D = -10, 0, 45$

max volume $= 36210\ cm^3$ ($= 0.03621\ m^3$)



$D \approx 28.6$ cm

$L \approx 38.6$ cm

$W \approx 32.8$ cm

THINK FURTHER

In the problem above, suppose the sum of the length, width, and depth was halved. Would the maximum volume also be halved? Explain.

Polynomial functions and their graphs can be used to solve real-world problems.

Example 1 Using the Graph of a Cubic Function to Solve a Problem

A piece of sheet metal 25 cm long and 18 cm wide is used to make a box with no lid. Equal squares of side length x centimetres are cut from the corners and the sides are folded up.

a) Write a polynomial function to represent the volume, V, of the box in terms of x.

b) Graph the function. What is the domain?

c) To the nearest cubic centimetre, what is the maximum volume of the box? What size of square should be cut out to create a box with this volume? To the nearest tenth of a centimetre, what are the dimensions of the box?

Check Your Understanding

1. A piece of cardboard 30 cm long and 25 cm wide is used to make a box with no lid. Equal squares of side length x centimetres are cut from the corners and the sides are folded up.

a) Write a polynomial function to represent the volume, V, of the box in terms of x.

the box? What size of square should be cut out to create a box with this volume? To the nearest tenth of a centimetre, what are the dimensions of the box?

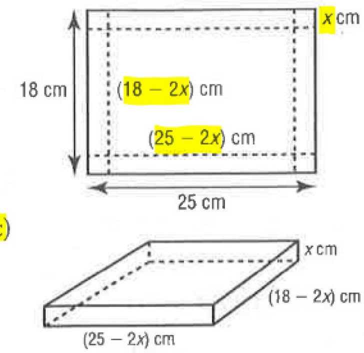
a) Write a polynomial function to represent the volume, V, of the box in terms of x.

$V(x) = x(25-2x)(30-2x)$

Domain: $0 < x$ 12.5

SOLUTION

a) Sketch a diagram.
The box has height x centimetres, width $(18 - 2x)$ centimetres, and length $(25 - 2x)$ centimetres.
So, a polynomial function that represents the volume, V, of the box is: $V(x) = x(18 - 2x)(25 - 2x)$



b) Graph the function. What is the domain?

c) To the nearest cubic centimetre, what is the maximum volume of the box? What size of square should be cut out to create a box with this volume? To the nearest tenth of a centimetre, what are the dimensions of the box?

b) Use a graphing calculator. Enter the equation:
$y = x(18 - 2x)(25 - 2x)$
The dimensions of the box are positive.
The sheet metal has width 18 cm.
So, the side length of a square cut from each corner must be less than $\frac{18 \text{ cm}}{2}$, or 9 cm.
So, the domain is: $0 < x < 9$
Use these window settings:

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WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=1000
 Yscl=200
 Xres=1
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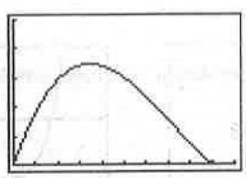
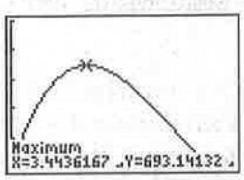


c) To determine the maximum volume, press: [2nd] [TRACE] [4], then use the arrow keys to move the cursor to determine the approximate coordinates of the local maximum point.



So, the maximum volume of the box is approximately 693 cm^3.
This occurs when each square that is cut out has a side length of approximately 3.4 cm.
The approximate dimensions of the box are:
Height: 3.4 cm
Width: 18 cm − 2(3.4 cm) = 11.2 cm
Length: 25 cm − 2(3.4 cm) = 18.2 cm
So, for maximum volume, the dimensions of the box are approximately 18.2 cm by 11.2 cm by 3.4 cm.

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Check Your Understanding

2. Clara and 3 friends were born on March 11. Lesley is 5 years younger than Clara. Mike is 2 years younger than Clara. Thomas is 3 years older than Clara. On March 11, 2011, the product of their ages was 61 136 greater than the sum of their ages. How old was Clara and each friend on that day?

Clara's age = x
Lesley's age = $x-5$
Mike's age = $x-2$
Thomas' age = $x+3$
sum of ages:
$x+x-5+x-2+x+3=4x-4$
$4x-4+61136=x(x-5)(x-2)(x+3)$ (61132)
$0=x(x-5)(x-2)(x+3)-4x-61132$

Clara was 17 years old.
Lesley was 12, Mike was 15, and Thomas was 20.

Example 2 Using the Graph of a Quartic Function to Solve a Problem

Leo and 3 friends each have birthdays on December 13. Sanda is 3 years younger than Leo. Leo is 4 years younger than Vince. Hunter is 1 year older than Leo. On December 13, 2010, the product of their ages was 54 658 greater than the sum of their ages. How old was Leo and each friend on that day?

SOLUTION

Let Leo's age in years be x.
Then, Sanda's age in years is $x - 3$, Vince's age in years is $x + 4$, and Hunter's age in years is $x + 1$. The sum of their ages is:
$x + (x - 3) + (x + 4) + (x + 1) = 4x + 2$
Sum of ages + 54 658 = product of ages
So, $4x + 2 + 54\,658 = x(x - 3)(x + 4)(x + 1)$
$0 = x(x - 3)(x + 4)(x + 1) - 4x - 54\,660$
Enter the equation $y = x(x - 3)(x + 4)(x + 1) - 4x - 54\,660$ into a graphing calculator. Graph the function using these window settings:

```
WINDOW
 Xmin=-30
 Xmax=30
 Xscl=5
 Ymin=-70000
 Ymax=60000
 Yscl=10000
 Xres=1
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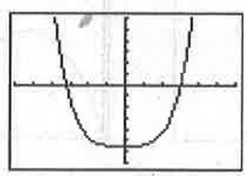
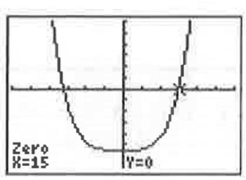


Since age cannot be negative, the positive x-intercept of the graph represents Leo's age.
To determine the positive x-intercept, press: 2nd TRACE 2, then use the arrow keys. The x-intercept is 15.
So, Leo was 15 years old on that day.



The ages of the other friends, in years, were: Sanda: $15 - 3 = 12$
Vince: $15 + 4 = 19$
Hunter: $15 + 1 = 16$

The ages of the other friends, in years, were: Sanda: 15 − 3 = 12
Vince: 15 + 4 = 19
Hunter: 15 + 1 = 16
On December 13, 2010, Leo was 15, Sanda was 12, Vince was 19, and Hunter was 16.

Assignment: # 1, 4a, 5ab, 6a, 7*, 9*, 10a

* requires graphing technology