

## 1.4 Negative and Zero Exponents

We know that exponents are used to simplify repeated multiplication expressions. For example,

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$$

But what do expressions like  $2^{-5}$  or  $8^0$  mean? Let's work backwards to figure this out.

$2^3$	$2 \times 2 \times 2 = 8$	
$2^2$	$2 \times 2 = 4$	$\downarrow \div 2$
$2^1$	2	$\downarrow \div 2$
$2^0$	1	$\downarrow \div 2$
$2^{-1}$	$\frac{1}{2}$	$\downarrow \div 2$
$2^{-2}$	$\frac{1}{4}$	$\downarrow \div 2 = \frac{1}{2^2} = \frac{1}{2 \times 2}$
$2^{-3}$	$\frac{1}{8}$	$\downarrow \div 2 = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2}$

$$x^{-m} = \frac{1}{x^m} ; \frac{1}{x^{-m}} = x^m$$

\*any power with exponent 0 is equal to 1

$$5^0 = 1 \quad a^0 = 1$$

$$(871 \times \frac{1}{41} + 3)^0 = 1$$

Rewrite each power with a positive exponent, then evaluate.

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

$$8^{-1} = \frac{1}{8^1} = \frac{1}{8}$$

$$3^0 = 1$$

exponent only applies to the 4  
 $\downarrow$   
 $-4^0 = -1$

$$(-2)^{-5} = \frac{1}{(-2)^5} = \frac{1}{-32}$$

$$(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$$

$$(-4)^0 = 1$$

$$-9^{-2} = -\frac{1}{9^2} = -\frac{1}{81}$$

$$\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = \frac{8}{1} = 8$$

$$\left(-\frac{3}{4}\right)^{-1} = \left(-\frac{4}{3}\right)^1 = -\frac{4}{3}$$

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

$$\left(-\frac{2}{3}\right)^{-3} = \left(-\frac{3}{2}\right)^3 = -\frac{27}{8}$$

$$5^{-1} + (6-2)^0 = \frac{1}{5} + 1 = 1\frac{1}{5}$$

Write the fraction  $\frac{1}{16}$  as a power of 2.

$$\frac{1}{16} = \frac{1}{2^4}$$

Simplify and rewrite each with only positive exponents.

$$a^{-4} \times a^{-3} = a^{-7} = \frac{1}{a^7}$$

$$6x^2 \div 2x^7 = 3x^{-5} = \frac{3}{x^5}$$

$$\frac{y^6}{2y^{-5}} = \frac{y^6 \cdot y^5}{2} = \frac{y^{11}}{2}$$

$$(-2x)^{-3} = \frac{1}{(-2x)^3} = \frac{1}{-8x^3}$$

$$\frac{8a^{-5}}{4b^{-3}} = \frac{2b^3}{a^5}$$

$$\begin{aligned} \frac{(5p)^{-2}}{5q^4} &= \frac{1}{5q^4(5p)^2} \\ &= \frac{1}{5q^4(25p^2)} \\ &= \frac{1}{125p^2q^4} \end{aligned}$$

**Practise:**

Simplify and rewrite each expression using only positive exponents.

$$3^{-2} =$$

$$14^{-1} =$$

$$-(-2)^0 =$$

$$(-4)^{-3} =$$

$$-5^{-2} =$$

$$\left(\frac{3}{4}\right)^{-2} =$$

$$\left(-\frac{7}{12}\right)^{-1} =$$

$$\left(-\frac{5}{8}\right)^0 =$$

$$\left(-\frac{5}{2}\right)^{-3} =$$

$$-\left(\frac{1}{2}\right)^{-6} =$$

$$(4^3)(4^{-5}) =$$

$$\frac{3^{-4}}{3^{-2}} =$$

$$\frac{12^3}{12^7} =$$

$$\left(\frac{8^{-1}}{8^0}\right)^3 =$$

$$(5^4)^{-2} =$$

$$\frac{1}{s^2t^{-6}} =$$

$$\frac{8t}{t^{-3}} =$$

$$\left(\frac{n^4}{n^{-4}}\right)^{-3} =$$

$$[(xy^4)^{-3}]^{-2} =$$

$$[(h^7)(h^{-2})]^{-2} =$$