1.2 Factoring Polynomials

PC12 1.2 Factoring Polynomials Name: _____

Factor each polynomial.

$6x^2 + 7x - 3$ $(3x-1)(2x+3)$

$2x^3 - 3x^2 - 14x$ $x(2x-7)(x+2)$

$49x^2 - 36y^4$ $(7x+6y^2)(7x-6y^2)$

Can you use the same method as above to factor $2x^3 - 9x^2 + 7x + 6$? No!

Consider the polynomial $P(x) = 5x^3 - 2x^2 + 8x - 48 = (x-2)(5x^2 + 8x + 24)$. What is the value of $P(2)$?

$x-2 = 0$, $x = 2$

$\therefore P(2) = 0$

Consider the polynomial $P(x) = 3x^3 + 2x^2 - x - 6 = (x+2)(3x^2 - 4x + 7) - 20$. What is the value of $P(-2)$?

$x+2 = 0$, $x = -2$

$P(-2) = -20$

Remainder Theorem

When a polynomial, $P(x)$, is divided by a binomial, $x - a$, $a \in Z$, the remainder is $P(a)$.

Example: Determine the remainder when $2x^4 - 5x^3 - 5x^2 + 5x + 3$ is divided by a) $x - 3$ and b) $x + 2$.

a) $P(3) = 2(3)^4 - 5(3)^3 - 5(3)^2 + 5(3) + 3$

$= 162 - 135 - 45 + 15 + 3$

$= 0$ The remainder is 0.

b) $P(-2) = 2(-2)^4 - 5(-2)^3 - 5(-2)^2 + 5(-2) + 3$

$= 32 + 40 - 20 - 10 + 3$

$= 45$ The remainder is 45.

The special case of the remainder theorem in which the remainder is 0 is called the factor theorem.

Factor Theorem

For $a \in Z$, $x - a$ is a factor of the polynomial $P(x)$ if $P(a) = 0$.

Example: Which binomials are factors of $x^3 - 6x^2 + 5x + 12$? a) $x + 1$ b) $x - 3$ c) $x - 4$ d) $x + 4$

a) $P(-1) = (-1)^3 - 6(-1)^2 + 5(-1) + 12 = 0$

b) $P(3) = (3)^3 - 6(3)^2 + 5(3) + 12 = 0$

c) $P(4) = (4)^3 - 6(4)^2 + 5(4) + 12 = 0$

d) $P(-4) = (-4)^3 - 6(-4)^2 + 5(-4) + 12 = -168$

$x+1$, $x-3$, and $x-4$ are factors.

Factor Property

If $x - a, a \in Z$, is a factor of a polynomial, then a is a factor of the constant term in the polynomial.

↳ term without a variable

Example: Factor fully: $2x^3 - 9x^2 + 7x + 6$

possible factors/values of "a": $\pm 1, \pm 2, \pm 3, \pm 6$

$P(1) = 2(1)^3 - 9(1)^2 + 7(1) + 6 = 6$

$P(-1) = 2(-1)^3 - 9(-1)^2 + 7(-1) + 6 = -12$

$P(2) = 2(2)^3 - 9(2)^2 + 7(2) + 6 = 0 \rightarrow x - 2$ is a factor

-2	2	-9	7	6
		-4	10	6
	2	-5	-3	0

$\rightarrow P(x) = (x-2)(2x^2 - 5x - 3)$

$P(x) = (x-2)(2x+1)(x-3)$

PC12 1.2 Practice: Name: _____

1. Determine the remainder when each polynomial is divided by $x - 2$.

a) $x^2 - 7x + 11$ b) $2x^3 - 3x^2 - 6x + 8$

2. When $2x^3 + kx^2 - 3x + 2$ is divided by $x - 2$, the remainder is 4. Determine the value of k.

3. Fully factor the polynomial $x^3 + 6x^2 + 3x - 10$.

4. Which values of $a, a \in Z$, should be chosen to test for binomial factors of the form $x - a$ of the polynomial $x^4 + 3x^3 - 8x^2 - 12x + 16$?

5. What value of b will ensure $x + 3$ is a factor of $bx^3 - 2x^2 + x - 6$?

6. Determine whether $x + b$ is a factor of $(x + b)^5 + (x + p)^2 + (b - p)^5, b, p \in R$.